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Modelling the risk of airborne infectious disease using exhaled air

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HIGHLIGHTS

- Quantified rebreathed air with high airborne infectious diseases transmission risk.
- We developed a mathematical model that predicts the risk of airborne infectious diseases.
- The model uses deposition fraction, particle survival and mortality rates but not quanta.
- The model works in multiple environments obeying infectious particles threshold level.
- The model matches individual and environment without limitations of uniformly mixed air.

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ABSTRACT

In this paper we develop and demonstrate a flexible mathematical model that predicts the risk of airborne infectious diseases, such as tuberculosis under steady state and non-steady state conditions by monitoring exhaled air by infectors in a confined space. In the development of this model, we used the rebreathed air accumulation rate concept to directly determine the average volume fraction of exhaled air in a given space. From a biological point of view, exhaled air by infectors contains airborne infectious particles that cause airborne infectious diseases such as tuberculosis in confined spaces. Since not all infectious particles can reach the target infection site, we took into account that the infectious particles that commence the infection are determined by respiratory deposition fraction, which is the probability of each infectious particle reaching the target infection site of the respiratory tracts and causing infection. Furthermore, we compute the quantity of carbon dioxide as a marker of exhaled air, which can be inhaled in the room with high likelihood of causing airborne infectious disease given the presence of infectors. We demonstrated mathematically and schematically the correlation between TB transmission probability and airborne infectious particle generation rate, ventilation rate, average volume fraction of exhaled air, TB prevalence and duration of exposure to infectors in a confined space.

1. Introduction

Endemic airborne infectious diseases, such as tuberculosis (TB) are transmitted in multiple congregate locations in the presence of infectors and with a low per person ventilation rate (Wells, 1955; Rudnick and Milton, 2003; Andrews et al., 2012; Richardson et al., 2014; Gammaitoni and Nucci, 1997). A high concentration of indoor rebreathed air by infectors is considered to be potentially harmful since it may contain airborne infectious particles from infectors, which could result in spread of airborne infectious diseases, such as TB (Emmerich and Persily, 2001; Murray et al., 2011; Richardson et al., 2014; Li et al., 2007). Carbon dioxide has been used as a measure of indoor air quality based on the concept that people emit carbon dioxide at a rate dependent on body mass and physical activity and that indoor carbon dioxide levels are determined by fresh air clearance (Emmerich and Persily, 2001; Persily, 1997). A room with environmental carbon dioxide concentration has about 400 ppm, but as people occupy it, the concentration of exhaled air starts to increase, depending on the ventilation rate per person, room volume, and the number of people in the room (Emmerich and Persily, 2001; Lygizos et al., 2013; Persily, 1997). This is because people in the room contribute to the increase in rebreathed air depending on their oxygen consumption, respiratory quotient and physical activities (Emmerich and Persily, 2001; Persily, 1997). When the concentration of exhaled air increases in the room with infectors present, the probability of susceptible individuals acquiring airborne infectious diseases also increases (Richardson et al., 2014). This is because exhaled air from infectious individuals usually contains airborne infectious particles within droplet nuclei that may remain

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airborne for prolonged periods and when inhaled may result in new infection of a susceptible individual (Wells, 1955). The progression from infection to TB disease depends on a number of different factors, including the state of the host immune system, host genetics, and the virulence of the infecting strain of *Mycobacterium tuberculosis* (Mtb) (Wells, 1955; Rieder, 1999a,b).

Respiratory activities such as talking, coughing, sneezing and singing could contribute to respiratory particle production (Loudon and Roberts, 1967). When airborne infectious particles are inhaled by a susceptible individual, only a fraction of the inhaled infectious particles may successfully reach the target infection site in the respiratory tract, as infectious particles with different sizes have different deposition fractions in different anatomical regions of the respiratory tract (Beggs et al., 2003; Sze To and Chao, 2010). For example, infectious particles with a critical size range from 1 μm to 5 μm have a higher probability of reaching and depositing on the alveolar region (Noakes and Sleigh, 2009; Rieder, 1999b) than those with sizes >5 μm, which are trapped in the upper respiratory tract. This implies that not all inhaled airborne infectious particles will reach or be retained at the target infection site. Thus, when assessing the risk of airborne infectious disease, the respiratory deposition fraction of airborne infectious particles must be taken into consideration. In this paper, we develop a flexible mathematical model that predicts the risk of airborne infectious diseases, such as TB under non-steady state and steady state conditions by monitoring exhaled air by infectors in a confined space. We start by describing the quantity of rebreathed air in an occupied room with the presence of infectors, which is inhaled by susceptible individuals with a high probability of acquiring airborne infectious diseases as demonstrated in the following section.

2. Quantity of rebreathed air inhaled in the room required to induce infection

Generally, elevated indoor carbon dioxide concentration is determined by the rate of exhaled air production and per person ventilation (Emmerich and Persily, 2001; Murray et al., 2011; Persily, 1997). As exhaled air from an infectious individual contains airborne infectious particles, carbon dioxide levels may be used as a surrogate for exhaled air (Emmerich and Persily, 2001; Richardson et al., 2014; Li et al., 2007; Persily, 1997; Wood et al., 2014). Exhaled air contains about 40,000 ppm of carbon dioxide compared with almost 400 ppm of carbon dioxide in the environmental air (Emmerich and Persily, 2001; Rudnick and Milton, 2003; Richardson et al., 2014). We make an assumption that an indoor space, such as a classroom of volume, V, starts the day with environmental carbon dioxide concentration, C_E, which is about 400 ppm and it becomes occupied by a number of people, n. This implies that the level of exhaled air concentration that might contain airborne infectious particles given the presence of infectors will start to increase in the room, depending on the ventilation rate, Q, and number of people in the room. We assume that people in the room will contribute equally to the generation of carbon dioxide as a marker of exhaled air. The fundamental equation for exhaled air accumulation rate in the room with environmental carbon dioxide, which then becomes occupied, is equal to the rate of exhaled air generated by occupants plus environmental carbon dioxide rate, minus exhaled air removed by ventilation rate:

$$V\frac{dC}{dt} = npC_a + QC_E - QC \tag{1}$$

where C is the indoor exhaled air concentration (ppm), p is the breathing rate (L/s) for each person in the room and C_a is the carbon dioxide fraction contained in breathed air.

To determine the quantity of exhaled air accumulated in the room with respect to elapsed time, we arrange Eq. (1) and integrate from C_E (which is the environmental carbon dioxide in the room) to $C_{(t)}$ (which is the total sampled exhaled air in the room) and elapsed time from 0 to t as follows:

$$\int_{C_E}^{C_{(t)}} \frac{dC}{npC_a + QC_E - QC} = \frac{1}{V}\int_0^t dt \tag{2}$$

After integrating Eq. (2) by application of integration rules, we obtain the following equation:

$$\ln\left[\frac{npC_a + QC_E - QC_{(t)}}{npC_a}\right] = -\frac{Qt}{V} \tag{3}$$

Simplifying Eq. (3) further by applying logarithmic and exponential rules, we obtain the mathematical model for sampled exhaled air accumulated in a room that started with environmental carbon dioxide, and then became occupied by a number of people:

$$C_{(t)} = C_E + \frac{npC_a}{Q}\left[1 - e^{-(QT/V)}\right] \tag{4}$$

where t is the time spent in the room.

Note that Eq. (4) holds only under non-steady-state conditions. However, as time, t, approaches infinity, the concentration, $C_{(t)}$, approaches the steady-state value, $C(s)$, and the time required to reach steady-state depends on the value of outdoor air exchange rate, Q/V, implying that the higher the value, the less the time required to reach the steady-state and Eq. (4) becomes

$$C(s) = C_E + \frac{npC_a}{Q} \tag{5}$$

Eq. (5) demonstrates that as the ventilation rate decreases in the buildings, the concentration of exhaled air that may contain airborne infectious particles, given the presence of infectors, increases.

Eq. (4) is demonstrated in Fig. 1, which shows that when the room becomes occupied the level of exhaled air increases as time elapses in the space. An experiment was conducted in a room of 75 m^3 while regulating the rate of ventilation from $Q=3$ L/s to $Q=4$ L/s, and measuring the quantity of carbon dioxide as a marker of rebreathed air. We noted that when the ventilation rate decreases, the concentration of exhaled air increases depending on the number of occupants and room ventilation rate.

We compute the total amount of exhaled air in the room, demonstrated in Fig. 1, as follows: Let $Q_{ABCDA} = Q_1$ and $Q_{CDEFC} = Q_2$. The horizontal line indicates environmental carbon dioxide ($C_E = 400$ ppm), which was assumed to be in the room before the entrance of occupants. Taking the summation integral of *ABCDA* and *CDEFC*, we obtain the total quantity of rebreathed air that was inhaled by each person present in the room for 1 h when ventilation rates were $Q_1 = 3$ L/s and $Q_2 = 4$ L/s, respectively. If T denotes elapsed time, the quantity of rebreathed air, $C_{(T)}$, which is generated in the room can be computed by taking the summation integral

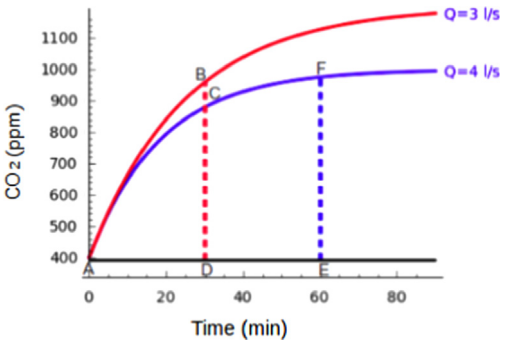

Fig. 1. The level of exhaled air in a room, which started a day with environmental carbon dioxide, then became occupied by a number of people.