Journal of Theoretical Biology ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/yjtbi

Evolutionary performance of zero-determinant strategies in multiplayer games

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H I G H L I G H T S

- We explore the evolution of direct reciprocity in groups of n players.
- We show why it is instructive to consider zero-determinant (ZD) strategies.
- ZD strategies include AllD, AllC, Tit-for-Tat, extortionate and generous strategies.
- In small groups, generosity allows the evolution of cooperation.
- In large groups, cooperation is unlikely to evolve.

A R T I C L E I N F O

Article history:
Received 29 December 2014
Received in revised form
12 March 2015
Accepted 24 March 2015

Keywords:
Cooperation
Repeated games
Zero-determinant strategies
Evolutionary game theory
Public goods game

A B S T R A C T

Repetition is one of the key mechanisms to maintain cooperation. In long-term relationships, in which individuals can react to their peers' past actions, evolution can promote cooperative strategies that would not be stable in one-shot encounters. The iterated prisoner's dilemma illustrates the power of repetition. Many of the key strategies for this game, such as *ALLD*, *ALLC*, Tit-for-Tat, or generous Tit-for-Tat, share a common property: players using these strategies enforce a linear relationship between their own payoff and their co-player's payoff. Such strategies have been termed zero-determinant (ZD). Recently, it was shown that ZD strategies also exist for multiplayer social dilemmas, and here we explore their evolutionary performance. For small group sizes, ZD strategies play a similar role as for the repeated prisoner's dilemma: extortionate ZD strategies are critical for the emergence of cooperation, whereas generous ZD strategies are important to maintain cooperation. In large groups, however, generous strategies tend to become unstable and selfish behaviors gain the upper hand. Our results suggest that repeated interactions alone are not sufficient to maintain large-scale cooperation. Instead, large groups require further mechanisms to sustain cooperation, such as the formation of alliances or institutions, or additional pairwise interactions between group members.

1. Introduction

One of the major questions in evolutionary biology is why individuals cooperate with each other. Why are some individuals willing to pay a cost (thereby decreasing their own fitness) in order to help someone else? During the last decades, researchers have proposed several mechanisms that are able to explain why cooperation is abundant in nature (Nowak, 2006; Sigmund, 2010). One such mechanism is repetition: if I help you today, you may help me tomorrow (Trivers, 1971). Among humans, this logic of reciprocal giving has been documented in numerous behavioral experiments (e.g., Wedekind and Milinski, 1996; Keser and van Winden, 2000; Fischbacher et al., 2001; Dreber et al., 2008; Grujic et al., 2015). Moreover, it has also been suggested that direct reciprocity is at work in several other species, including vampire bats (Wilkinson, 1984), sticklebacks (Milinski, 1987), blue jays (Stephens et al., 2002), and zebra finches (St. Pierre et al., 2009). From a theoretical viewpoint, these observations lead to the question under which circumstances direct reciprocity evolves, and which strategies can be used to sustain mutual cooperation.

The main model to explore these questions is the iterated prisoner's dilemma, a stylized game in which two individuals repeatedly decide whether they cooperate or defect (Rapoport and

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http://dx.doi.org/10.1016/j.jtbi.2015.03.032
0022-5193/© 2015 Published by Elsevier Ltd.

Please cite this article as: Hilbe, C., et al., Evolutionary performance of zero-determinant strategies in multiplayer games. J. Theor. Biol. (2015), http://dx.doi.org/10.1016/j.jtbi.2015.03.032

Chammah, 1965; Doebeli and Hauert, 2005). The payoffs of the game are chosen such that mutual cooperation is preferred over mutual defection, but each individual is tempted to defect at the expense of the co-player. Theoretical studies have highlighted several successful strategies for this game (Axelrod and Hamilton, 1981; Molander, 1985; Kraines and Kraines, 1989; Nowak and Sigmund, 1992, 1993b). Evolution often occurs in dynamical cycles (Boyd and Lorberbaum, 1987; Nowak and Sigmund, 1993a; van Veelen et al., 2012): unconditional defectors (*ALLD*) can be invaded by reciprocal strategies like Tit-for-Tat (*TFT*), which in turn often catalyze the evolution of more cooperative strategies like generous Tit-for-Tat (*gTFT*) and unconditional cooperators (*ALLC*). Once *ALLC* is common, *ALLD* can reinvade, thereby closing the evolutionary cycle (Nowak and Sigmund, 1989; Imhof et al., 2005; Imhof and Nowak, 2010).

The above mentioned strategies for the iterated prisoner's dilemma share an interesting mathematical property: they enforce a linear relationship between the players' payoffs in an infinitely repeated game (Press and Dyson, 2012). For example, when player 1 adopts the strategy Tit-for-Tat, the players' payoffs π_j will satisfy the equation $\pi_1-\pi_2=0$, irrespective of player 2's strategy. Similarly, when player 1 adopts *ALLD*, payoffs will satisfy $c\pi_1+b\pi_2=0$ (where c and b denote the cost and the benefit of cooperation,respectively; this version of the prisoner's dilemma is sometimes called the donation game,see e.g. Sigmund, 2010). Finally, when player 1 applies *gTFT*, the enforced payoff relation becomes $\pi_2=b$. Strategies that enforce such linear relationships between payoffs have been called zero-determinant strategies, or ZD strategies (this name is motivated by the fact that these strategies let certain determinants vanish, see Press and Dyson, 2012). After Press and Dyson's discovery, several studies have explored how ZD strategies for the repeated prisoner's dilemma fare in an evolutionary context (Akin, 2013; Stewart and Plotkin, 2012, 2013; Hilbe et al., 2013a,b; Adami and Hintze, 2013; Szolnoki and Perc, 2014a,b; Chen and Zinger, 2014), and in behavioral experiments (Hilbe et al., 2014a).

Zero-determinant strategies are not confined to pairwise games; they also exist in the iterated public goods game (Pan et al., 2014), and in fact in any repeated social dilemma, with an arbitrary number of involved players (Hilbe et al., 2014b). In this way, it has become possible to identify the multiplayer-game analogues of the above mentioned strategies. For example, the multiplayer-version of *TFT* in a repeated public goods game is *proportional Tit-for-Tat* (*pTFT*): if j of the other group members cooperated in the previous round, then a *pTFT*-player cooperates with probability $j/(n-1)$ in the next round, with n being the size of the group. Herein, we will explore the role of these recently discovered multiplayer ZD strategies for the evolution of cooperation.

We consider two evolutionary scenarios. First, we consider a conventional setup, in which the members of a well-mixed population are engaged in a series of repeated public goods games, and where successful strategies reproduce more often. In line with previous studies (Boyd and Richerson, 1988; Hauert and Schuster, 1997; Grujic et al., 2012), our simulations confirm that the prospects of cooperation depend on the size of the group. Small groups promote generous ZD strategies that allow for high levels of cooperation, whereas larger groups favor the emergence of selfish ZD strategies such as *ALLD*. For our second evolutionary scenario, we consider a player with a fixed ZD strategy whose co-players are allowed to adapt their strategies over time. Similar to the case of the repeated prisoner's dilemma (Press and Dyson, 2012; Chen and Zinger, 2014), the resulting group dynamics then depends on the applied ZD strategy of the focal player. But also here, the possibilities of a single player to generate a positive group dynamics diminishes with group size, irrespective of the strategy applied by the focal player.

Taken together, these results suggest that larger groups make it more difficult to sustain cooperation. In the discussion, we will thus argue that there are three potential mechanisms that can help individuals solving their multiplayer social dilemmas: they can either provide additional incentives on a pairwise basis (Rand et al., 2009; Rockenbach and Milinski, 2006); they can coordinate their actions and form alliances (Hilbe et al., 2014b); or they can implement central institutions which enforce mutual cooperation (Ostrom, 1990; Sigmund et al., 2010; Sasaki et al., 2012; Cressman et al., 2012; Traulsen et al., 2012; Zhang and Li, 2013; Schoenmakers et al., 2014).

2. Model

2.1. Iterated multiplayer dilemmas and memory-one strategies

In the following, we consider a group of n individuals, which is engaged in a repeated multiplayer dilemma. In each round of the game, players can decide whether to cooperate (C) or to defect (D). The payoffs in a given round depend on the player's own decision, and on the number of cooperators among the remaining group members. That is, in a round in which j of the other $n-1$ group members cooperate, the focal player receives a_j for cooperation, and b_j for defection (see also Table 1). We suppose that the multiplayer game takes the form of a social dilemma, such that payoffs satisfy the following three conditions (see also Kerr et al., 2004): (a) individuals prefer their co-players to be cooperative, $a_{j+1}\ge a_j$ and $b_{j+1}\ge b_j$ for all j; (b) within a mixed group, defectors outperform cooperators, $b_{j+1}>a_j$ for all j; (c) mutual cooperation is favored over mutual defection, $a_{n-1}>b_0$. Several well-known examples of multiplayer games satisfy these criteria, including the public goods game (see e.g. Ledyard, 1995), the volunteer's dilemma (Diekmann, 1985; Archetti, 2009), or the collective-risk dilemma (Milinski et al., 2008; Santos and Pacheco, 2011; Abou Chakra and Traulsen, 2014).

We assume that the multiplayer game is repeated, such that the group members face the same dilemma situation over multiple rounds. Herein, we will focus on infinitely repeated games, but the theory of ZD strategies can also be developed for games with finitely many rounds, or when future payoffs are discounted (Hilbe et al., 2014a, 2015). In repeated games, players can react on their co-players' previous behavior. In the simplest case, players only consider the outcome of the last round, that is, they apply a so-called memory-one strategy. Memory-one strategies consist of two parts: a rule that tells the player what to do in the first round, and a rule for what to do in all subsequent rounds, depending on the previous round's outcome. In infinitely repeated games, the first-round play can typically be neglected (see also Appendix A.1). In that case, memory-one strategies can be written as a vector $\mathbf{p}=(p_{C,n-1},\dots,p_{C,0};p_{D,n-1},\dots,p_{D,0})$. The entries $p_{S,j}$ correspond to

Table 1
Payoff table for multiplayer games with n group members (see also Gokhale and Traulsen, 2010; van Veelen and Nowak, 2012; Gokhale and Traulsen, 2014; Peña et al., 2014; Du et al., 2014). The payoff of a player depends on the player's own action, and on the number of cooperating co-players. As an example of a multiplayer dilemma, we will discuss linear public good games. There, cooperators contribute an amount $c>0$ to a common pool. Total contributions to the common pool are multiplied by a factor r with $1<r<n$, and evenly shared among all group members. Thus, the payoff of a cooperator is $a_j=rc(j+1)/n-c$, whereas the payoff of a defector is $b_j=rcj/n$.

Number of cooperating co-players	$n-1$	$n-2$	…	0
Payoff for cooperation	a_{n-1}	a_{n-2}	…	a_0
Payoff for defection	b_{n-1}	b_{n-2}	…	b_0

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