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Is trabecular bone permeability governed by molecular ordering-induced fluid viscosity gain? Arguments from re-evaluation of experimental data in the framework of homogenization theory

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HIGHLIGHTS

- Poiseuille flow in pore channels is upscaled to overall trabecular bone permeability.
- Homogenization schemes from micromechanics are adapted for transport modeling.
- Homogenized permeability follows analytical formula extending Kozeny–Carman relation.
- Increased viscosity of polarized fluids appears as essential.

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ABSTRACT

It is generally agreed on that trabecular bone permeability, a physiologically important quantity, is governed by the material's (vascular or intertrabecular) porosity as well as by the viscosity of the pore-filling fluids. Still, there is less agreement on how these two key factors govern bone permeability. In order to shed more light onto this somewhat open issue, we here develop a random homogenization scheme for upscaling Poiseuille flow in the vascular porosity, up to Darcy-type permeability of the overall porous medium "trabecular bone". The underlying representative volume element of the macroscopic bone material contains two types of phases: a spherical, impermeable extracellular bone matrix phase interacts with interpenetrating cylindrical pore channel phases that are oriented in all different space directions. This type of interaction is modeled by means of a self-consistent homogenization scheme. While the permeability of the bone matrix equals to zero, the permeability of the pore phase is found through expressing the classical Hagen–Poiseuille law for laminar flow in the format of a "micro-Darcy law". The upscaling scheme contains pore size and porosity as geometrical input variables; however, they can be related to each other, based on well-known relations between porosity and specific bone surface. As two key results, validated through comprehensive experimental data, it appears (i) that the famous Kozeny–Carman constant (which relates bone permeability to the cube of the porosity, the square of the specific surface, as well as to the bone fluid viscosity) needs to be replaced by an again porosity-dependent rational function, and (ii) that the overall bone permeability is strongly affected by the pore fluid viscosity, which, in case of polarized fluids, is strongly increased due to the presence of electrically charged pore walls.

1. Introduction

Trabecular bone permeability enables important physiological processes, such as bone remodeling or fracture healing. Namely, the latter processes are accelerated through increased blood pressure (Li et al., 1987; Qin et al., 2002); and it is known that permeability significantly affects fluid flow in the trabecular bone (Malandrino et al., 2009), this flow being often considered as important cell stimulus. Consequently, the aforementioned permeability properties need to be carefully mimicked when designing implant scaffolds (Nauman et al., 1999; Truscello et al., 2012). On the other hand, bone permeability defines the requirements for successful application of surgical techniques, such as vertebroplasty (Baroud et al., 2004); and it is also a key property governing diagnostic techniques, such as ultrasound propagation in bone (Buchanan and Gilbert, 2007; Grimes et al., 2012). This has led to

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Nomenclature

Abbreviations

A–P anterior–posterior
CFD computational fluid dynamics
exvas extravascular
hom homogenized
RVE representative volume element
S–I superior–inferior

Mathematical operators

d derivative operator
grad microscopic gradient
GRAD macroscopic gradient
div divergence
$\otimes$ dyadic product
$\int$ integration
$\partial/\partial s$ partial derivative with respect to variable s

Latin symbols

$\mathbf{A}_{pore}$ pressure gradient concentration tensor
a_i coefficients used in Eq. (7), $i=0,1,2,3,4,5$
$\mathbf{e}_i$ unit vector in Cartesian base frame, $i=1,2,3$
f_{pore} volume fraction of vascular pores
$\mathbf{k}$ microscopic permeability tensor
$\mathbf{K}_{int}^{exp}$ experimentally determined tensor of intrinsic permeabilities
K_{int}^{exp} experimentally determined intrinsic permeability tensor component
$\mathbf{K}_{int}^{hom}$ homogenized tensor of intrinsic permeabilities
K_{int}^{hom} intrinsic permeability tensor component from homogenization
$\mathbf{K}^{exp}$ experimentally determined macroscopic permeability tensor
K^{exp} experimentally determined macroscopic permeability tensor component
$\mathbf{K}^{hom}$ homogenized permeability tensor
K^{hom} homogenized permeability tensor component
$\mathbf{k}_{pore}$ permeability tensor of the pore space
l_{pore} pore length
$\mathcal{L}$ characteristic length of a structure built up by the material defined on the RVE
l_{RVE} characteristic length of RVE
p microscopic pressure
P macroscopic pressure
$\mathbf{P}$ inhomogeneity tensor
$\mathbf{P}_{exvas}$ inhomogeneity tensor for extravascular solid matrix
$\mathbf{P}_{pore}$ inhomogeneity tensor for vascular (intertrabecular) pore space
$\mathcal{P}$ characteristic length of the loading of a structure build up by the material defined on the RVE
r radial distance
R inner radius of cylindrical tube
s coordinate measuring along the longitudinal tube direction
S_V specific surface
$\mathbf{v}$ microscopic fluid velocity
V_{RVE} volume of RVE
$\mathbf{V}$ macroscopic velocity
v_{tube} velocity distribution across the tube cross section
v_{pore} mean velocity of fluid in the pore
$\mathbf{x}$ microscopic location vector
$\mathbf{1}$ identity tensor

Greek letters

α polar angle in cylindrical coordinates
α_1 polar angle in spherical coordinates
α_2 azimuthal angle in spherical coordinates
δ_{ij} Kronecker delta
η dynamic viscosity of fluid
ϑ Eulerian angle in Euclidean space
$\boldsymbol{\xi}$ location vector
ξ_i component of location vector, $i=\varphi, \vartheta$
φ Eulerian angle in Euclidean space
ψ potential function
ψ_{exvas} potential function for extravascular bone tissue
ψ_{pore} potential function for vascular (intertrabecular) pore space

intensive studies on bone permeability, through both experiments (Baroud et al., 2004; Nauman et al., 1999; Grimm and Williams, 1997) as well as theory and computation (Abdul Kadir and Syahrom, 2009; Teo and Teoh, 2012): The experimental studies have evidenced a surprisingly large variation of permeability properties; and the identification of power-law or logarithmic relationships between these properties and underlying physiological characteristics, such as porosity, has turned out as quite challenging (Nauman et al., 1999; Baroud et al., 2004). As one remedy to this somewhat unsatisfactory situation, several researchers have invested into precise representation of the pore morphology in computational fluid dynamics (CFD) simulations (Abdul Kadir and Syahrom, 2009; Syahrom et al., 2013; Teo et al., 2005; Teo and Teoh, 2012; Zeiser et al., 2008). However, even with detailed CFD simulations, the large specimen-specific variations in bone permeability cannot be fully explained – and the computational expenses needed for CFD simulations, even when being much cheaper than solid mechanics analyses, may render the realization of very many simulation results related to very many different trabecular microstructures and/or fluid properties, as quite challenging. In addition, CFD simulations of trabecular bone microstructures also require extensive micro-CT scanning activities, so as to provide the needed, detailed geometrical information – such activities may be limited, or even unfeasible in specific clinical settings. Hence, the question arises whether reliable identification of bone microstructure–permeability relations turns out as unachievable task, or whether a fundamentally changed viewpoint onto the problem might help to further elucidate the physical and microstructural origins of bone permeability. In this context, the present paper deals with the following research questions:

(I) The first question is related to deepening the physical understanding of the flow problem; reading: May the Poiseuille flow in the pore channels be governed by the electrically charged bone matrix surfaces inducing layered structuring of bipolar pore fluids – and therefore increase these fluids' viscosities as compared to the corresponding undisturbed bulk fluid states?
(II) Secondly, we strive for finding a compromise between the rather expensive CFD simulations requiring detailed microstructure information on the one hand, and the use of somewhat over-simplistic relations adopted from soil engineering (Carman, 1939; Kozeny, 1927) on the other hand. Therefore, we ask: Can the bone porosity–permeability relations be derived