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Evolution via imitation among like-minded individuals

Naoki Masuda

Department of Mathematical Informatics, The University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-8656, Japan

HIGHLIGHTS

- I study an evolutionary game model with idiosyncratic fitness.
- The model behaves differently from other models such as the bimatrix game.
- Polarization of strategies in different subpopulations often occurs.

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ABSTRACT

In social situations with which evolutionary game is concerned, individuals are considered to be heterogeneous in various aspects. In particular, they may differently perceive the same outcome of the game owing to heterogeneity in idiosyncratic preferences, fighting abilities, and positions in a social network. In such a population, an individual may imitate successful and similar others, where similarity refers to that in the idiosyncratic fitness function. I propose an evolutionary game model with two subpopulations on the basis of multipopulation replicator dynamics to describe such a situation. In the proposed model, pairs of players are involved in a two-person game as a well-mixed population, and imitation occurs within subpopulations in each of which players have the same payoff matrix. It is shown that the model does not allow any internal equilibrium such that the dynamics differs from that of other related models such as the bimatrix game. In particular, even a slight difference in the payoff matrix in the two subpopulations can make the opposite strategies to be stably selected in the two subpopulations in the snowdrift and coordination games.

1. Introduction

A basic assumption underlying many evolutionary and economic game theoretical models is that individuals are the same except for possible differences in the strategy that they select. In fact, a population of individuals involved in ecological or social interaction is considered to be heterogeneous. For example, different individuals may have different fighting abilities or endowments (Landau, 1951; Hammerstein, 1981; Maynard Smith, 1982; McNamara et al., 1999), occupy different positions in contact networks specifying the peers with whom the game is played (Szabó and Fáth, 2007; Jackson, 2008), or have different preferences over the objective outcome of the game. The last situation is succinctly represented by the Battle of the Sexes game in which a wife and husband prefer to go to watch opera and football, respectively, whereas their stronger priority is on going out together (Luce and Raiffa, 1957) (the Battle of the Sexes game here is different from the one that models conflicts between males and females concerning parental investment as described in Dawkins (1976), Schuster and Sigmund (1981), Maynard Smith (1982), Hofbauer and Sigmund (1988), and Hofbauer and Sigmund (1998). In behavioral game experiments, the heterogeneity of subjects is rather a norm than exceptions (e.g., Camerer, 2003). For example, some humans are cooperative in the public goods game and others are not (e.g., Fischbacher et al., 2001; Jacquet et al., 2012), and some punish non-cooperators more than others do (Fehr and Gächter, 2002; Dreber et al., 2008).

Evolution of strategies in such a heterogeneous population is the focus of the present paper. This question has been examined along several lines.

First, in theory of preference, it is assumed that individuals maximize their own idiosyncratic utilities that vary between individuals. The utility generally deviates from the fitness on which evolutionary pressure operates (e.g., Sandholm, 2001; Dekel et al., 2007; Alger and Weibull, 2012; Grund et al., 2013).

In fact, experimental evidence shows that individuals tend to imitate behavior of similar others in the context of diffusion of innovations (Rodgers, 2003) and health behavior (Centola, 2011). Also in the context of economic behavior described as games, individuals may preferentially imitate similar others because

E-mail address: masuda@mist.i.u-tokyo.ac.jp

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similar individuals are expected to be interested in maximizing similar objective functions. This type of behavior is not considered in previous preference models in which individuals can instantaneously maximize their own payoffs, and selection occurs on the basis of the fitness function common to the entire population. The model proposed in this study deals with evolutionary dynamics in which individuals in a heterogeneous population mimic successful and similar others. The similarity here refers to that in the idiosyncratic preference.

Second, evolution in heterogeneous populations has been investigated with the use of the evolutionary bimatrix game (Hofbauer and Sigmund, 1988, 1998; Weibull, 1995). A payoff bimatrix describes the payoff imparted to the two players in generally asymmetric roles. In its evolutionary dynamics, a population is divided into two subpopulations, pairs of individuals selected from the different subpopulations play the game, and selection occurs within each subpopulation. The population then has bipartite structure induced by the fixed role of individuals. However, the most generic population structure for investigating interplay of evolution via social learning and idiosyncratic preferences would be a well-mixed population without fixed roles of individuals.

Third, evolutionary game dynamics on heterogeneous social networks (Szabó and Fáth, 2007) is related to evolution in heterogeneous populations. In most of the studies on this topic, the payoff to an individual per generation is defined as the obtained payoff summed over all the neighboring individuals. Then, cooperation in social dilemma games is enhanced on heterogeneous networks (Santos and Pacheco, 2005; Durán and Mulet, 2005; Santos et al., 2006). In this framework, hubs (i.e., those with many neighbors) and non-hubs are likely to gain different payoffs mainly because of their positions in the contact network. In particular, if the payoff of a single game is assumed to be nonnegative, hubs tend to earn more than non-hubs simply because hubs have more neighbors than non-hubs by definition (Masuda, 2007). However, as long as the contact network is fixed, a non-hub player will not gain a large payoff by imitating the strategy of a successful hub neighbor. The number of neighbors serves as the resource of a player. Then, it may be more natural to assume that players imitate successful others with a similar number of neighbors.

Motivated by these examples, I examine evolutionary dynamics in which a player would imitate successful others having similar preferences or inhabiting similar environments. I divide the players into two subpopulations depending on the subjective perception of the result of the game; one may like a certain outcome of the game, and another may not like the same outcome. Imitation is assumed to occur within each subpopulation. However, the interaction occurs as a well-mixed population. I also assume that all the individuals have the same ability, i.e., no player is more likely to "win" the game than others.

2. Model

Consider a population comprising two subpopulations of players such that the payoff matrix depends on the subpopulation. The payoff is equivalent to the fitness in the present model. I call the game the subjective payoff game. Each player, independent of the subpopulation, selects either of the two strategies denoted by A and B. The case with a general number of strategies can be analogously formulated. The subjective payoff game and its replicator dynamics described in the following are a special case of the multipopulation game proposed before (Taylor, 1979; Schuster et al., 1981a) (for slightly different variants, see Maynard Smith, 1982; Hofbauer and Sigmund, 1988; Weibull, 1995).

The population is infinite, well-mixed, and consists of a fraction p $(0<p<1)$ of type X players and a fraction $1-p$ of type Y players. The subjective payoff matrices that an X player and a Y player perceive as row player are defined by

$$\begin{array}{c} \\ A \\ B \end{array}\!\!\begin{array}{c} \;\;A \quad\; B \\ \begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix} \end{array} \quad \text{and} \quad \begin{array}{c} \\ A \\ B \end{array}\!\!\begin{array}{c} \;\;A \quad\; B \\ \begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix} \end{array}, \tag{1}$$

respectively. It should be noted that the payoff that an X player, for example, perceives depends on the opponent's strategy (i.e., A or B) but not on the opponent's type (i.e., X or Y). The use of the two payoff matrices represents different idiosyncrasies in preferences in the two subpopulations. Alternatively, the payoff matrix differs by subpopulations because X and Y players have different tendencies to transform the result of the one-shot game (i.e., one of the four consequences composed of a pair of A and B) into the fitness. For example, X and Y players may benefit the most from mutual A and mutual B, respectively.

The fractions of X and Y players that select strategy A are denoted by x and y, respectively. The fractions of X and Y players that select strategy B are equal to $1-x$ and $1-y$, respectively. The payoffs to an X player with strategies A and B are given by

$$\pi_{X,A} = a_X[px+(1-p)y]+b_X[p(1-x)+(1-p)(1-y)] \tag{2}$$

and

$$\pi_{X,B} = c_X[px+(1-p)y]+d_X[p(1-x)+(1-p)(1-y)], \tag{3}$$

respectively. The payoff to a Y player is defined with X replaced by Y in Eqs. (2) and (3).

I assume that in the evolutionary dynamics, the players can only copy the strategies of peers in the same subpopulation. This assumption reflects the premise that the payoff in the present model is subjective such that the only comparison that makes sense is that between the players in the same subpopulation. The replicator dynamics of the subjective payoff game is then defined by

$$\begin{aligned}\dot{x} &= x[\pi_{X,A}-(x\pi_{X,A}+(1-x)\pi_{X,B})] = x(1-x)\{(a_X-c_X)[px+(1-p)y] \\ &\quad +(b_X-d_X)[p(1-x)+(1-p)(1-y)]\}\end{aligned} \tag{4}$$

and

$$\dot{y} = y(1-y)\{(a_Y-c_Y)[px+(1-p)y]+(b_Y-d_Y)[p(1-x)+(1-p)(1-y)]\}, \tag{5}$$

where $\dot{x}$ and $\dot{y}$ represent the time derivatives.

3. General results

3.1. Absence of internal equilibrium

If (x, y) is an internal equilibrium (i.e., $0<x,y<1$) of the replicator dynamics given by Eqs. (4) and (5), $(a_X-c_X)[px+(1-p)y]+(b_X-d_X)[p(1-x)+(1-p)(1-y)] = (a_Y-c_Y)[px+(1-p)y]+(b_Y-d_Y)[p(1-x)+(1-p)(1-y)] = 0$ must be satisfied. However, this is impossible unless a degenerate condition $(a_X-c_X)(b_Y-d_Y) = (a_Y-c_Y)(b_X-d_X)$ is satisfied. Therefore, for a generic pair of payoff matrices, the replicator dynamics does not have an internal equilibrium.

Three remarks are in order. First, the absence of internal equilibrium implies that the present dynamics does not allow limit cycles (Hofbauer and Sigmund, 1988, 1998). Second, the present result contrasts with that for a two-subpopulation dynamics in which the perceived payoff matrix depends on the opponent's subpopulation as well as on the focal player's subpopulation. In the latter case, an internal equilibrium or limit cycle can exist (Schuster et al., 1981a). Third, the present conclusion is