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Hidden patterns of reciprocity

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HIGHLIGHTS

- Reciprocity can help the evolution of cooperation.
- In the context of direct reciprocity there exist four second-order action rules which are able to promote cooperation.
- In the context of indirect reciprocity there exist four second-order assessment rules which are able to promote cooperation.
- The four action rules and the four assessment rules can be paired, and they show very similar patterns.
- These common patterns are based on the relationship to the punishment.

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ABSTRACT

Reciprocity can help the evolution of cooperation. To model both types of reciprocity, we need the concept of strategy. In the case of direct reciprocity there are four second-order action rules (Simple Tit-for-tat, Contrite Tit-for-tat, Pavlov, and Grim Trigger), which are able to promote cooperation. In the case of indirect reciprocity the key component of cooperation is the assessment rule. There are, again, four elementary second-order assessment rules (Image Scoring, Simple Standing, Stern Judging, and Shunning). The eight concepts can be formalized in an ontologically thin way we need only an action predicate and a value function, two agent concepts, and the constant of goodness. The formalism helps us to discover that the action and assessment rules can be paired, and that they show the same patterns. The logic of these patterns can be interpreted with the concept of punishment that has an inherent paradoxical nature.

Human societies are based on cooperation. The big question is what kind of mechanism or strategy can provide and maintain the mutual cooperation among individuals when there is a strong temptation on mutual defection. These situations can be modeled by Prisoners's Dilemma (PD) or Iterated PD (IPD) game (Axelrod, 1984). Reciprocity is a very important concept of social sciences, especially in sociology and anthropology (see the works of Mauss, 1990; Malinowski, 1950; Sahlins, 1972), but it is used within the evolutionary biology as well (Trivers, 1971; Alexander, 1987). In the last two decades reciprocity was explored by game theory, and many theories and simulations were born (Sigmund, 2010). Trivers introduced the concept of direct reciprocity (DR) as a mechanism promoting and maintaining cooperation between two players who know each other. The latter moment, the expectation of familiarity has a natural condition: direct reciprocity provides cooperation only among small numbers of somehow related, at least familiar individuals. The key component of the explanation is the action rule (ACR), and the key feature of the successful ACRs is the repetition (Sigmund, 2010).

Alexander has proposed that large-scale cooperation among humans can be explained with the help of the concept of indirect reciprocity (IR). After some early attempts (Boyd and Richerson, 1989) in 1998 Sigmund and Nowak developed a game theoretic formal model of indirect reciprocity. In order to explain IR we need to add a new component to the model of IR, an assessment rule (ASR). With the help of an ASR group members can change the reputation of individuals. In the context of indirect reciprocity players do not know each either, they do not know how the others behaved in the previous round and when individuals must decide about cooperation they can base their decision only on reputation information. There exists a very strong condition of the theories of reciprocity: actors have only two possibilities, they can only cooperate or defect. We can differentiate between two types of reciprocity: the one is positive reciprocity when the *agent* reciprocates something *Good* (*Cooperation*), the other is negative reciprocity when the *agent* repays something *Bad* (*Defection*).

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1. Direct reciprocity

The language I use is an ordinary first-order predicate logic with two complementing deontic logic operators. Formalizing the concept of ACR first we need is an action predicate: $\textsc{do}(agent, a, t_i)$ (Von Wright, 1963; Kanger and Kanger, 1966; Belnap et al., 2001). Here the category of action should have at least three parameters. First, there must be an *agent* who does or forbears something. In the context of reciprocity an *agent* can be *Ego* or *Alter*. The second parameter of the predicate refers to the content of the action, and the third parameter indicates the round of the repeated game environment. An action rule determines how the *agent* should decide for each round in an iterated game. In a second step we introduce a valuation/assessment function ($\textsc{value}(x, t)$), supposing a very simple completely Black & White world where things (x) at a given moment (t) can be assessed only as *Good* or *Bad* (where *Good* and *Bad* are undefined constants of our language)

$$\textsc{g}(x, t_i) \leftrightarrow \textsc{value}(x, t_i) = Good$$

$$\textsc{b}(x, t_i) \leftrightarrow \textsc{value}(x, t_i) = Bad$$

The two statements above can be expressed in another form. In a dichotomous, Black & White world it is true that

$$\neg(\textsc{value}(x, t_i) = Good) \leftrightarrow \textsc{value}(x, t_i) = Bad$$

or shortly

$$\textsc{b}(x, t_i) \leftrightarrow \neg\textsc{g}(x, t_i)$$

In the theories of reciprocity there are only two possible assessments of action: it can be *Good* or *Bad*. We can formalize it in the following way:

$$\textsc{do}(agent, a, t_i) \rightarrow (\textsc{value}(a, t_i) = Good \leftrightarrow \neg(\textsc{value}(a, t_i) = Bad))$$

or shortly

$$\textsc{do}(agent, a, t_i) \rightarrow (\textsc{g}(a, t_i) \leftrightarrow \neg\textsc{b}(a, t_i))$$

Based on the equations above we can define the category of *cooperation* ($\textsc{c}(agent, t_i)$), and the category of *defection* ($\textsc{d}(agent, t_i)$)

$$\textsc{c}(agent, t_i) \leftrightarrow \textsc{do}(agent, a, t_i) \wedge \textsc{value}(a, t_i) = Good$$

$$\textsc{d}(agent, t_i) \leftrightarrow \textsc{do}(agent, a, t_i) \wedge \textsc{value}(a, t_i) = Bad$$

The connection between the two types of action is obvious

$$\textsc{d}(agent, t_i) \leftrightarrow \neg\textsc{c}(agent, t_i)$$

The next question is what kind of strategies (action rules) can be found, and how they can be formalized. There are conditional and unconditional ACRs. An example of an unconditional strategy is *AllD*, the so-called always defector (willing defector) strategy. Similarly, for *AllC* we can use an always cooperator (willing cooperator) strategy. The unconditional ACRs are too simple, they could not win a game series. Theoretically the condition of a conditional ACR can be any state of affairs, but in this context the conditional ACRs' condition is always a former action. It has two consequences: first, the extension of the concept of condition is narrower here as it is usual; second, the category of conditional action rule is a higher-order concept. If we would like to grasp the prescriptive character of our action rules we have to apply the *obligatory* operator from the field of deontic logic. When it is obligatory to do something for an agent this fact can be expressed in the following way:

$$\mathbf{O}\textsc{do}(agent, a, t_i)$$

As a first step a simple first round sub-rule can be stated for all ACRs. In the first round cooperation is obligatory for both players. This is an exception rule, because it refers only the first step

$$\textsc{acr}(Ego, t_1) \leftrightarrow \mathbf{O}\textsc{c}(Ego, t_1)$$

It is true for all action rules, so it is unnecessary to take into consideration any further. The four action rules can be interpreted without this sub-rule. Let us see how.

(i) Maybe the most famous experiment was Axelrod's round-robin tournament (Axelrod, 1984), where different strategies competed in an Iterated Prisoner's Dilemma game (IPD). The winning rule was the well-known *Tit-for-tat* (*tft*) strategy proposed by Rapoport. This rule cooperates in the first step, and in all other rounds repeats its partner's previous action. This is the prototypical ACR of direct reciprocity. *Tit-for-tat* strategy is very old, it can be found everywhere in our history. We have lots of proverbs with the same (or similar) meaning. “An eye for an eye, a tooth for a tooth.” or “He who greets with a stick, will be answered with a club.” is for the negative reciprocity, “One good turn deserves another.” for positive reciprocity, and “Bread borrowed should be returned.” for both. Axelrod (1984) evaluated TFT as a nice, retaliating, forgiving, and non-envious strategy. The next simple formula shows how we can describe the main rule of reciprocity: ‘replicate your partner's moves’

$$\textsc{tft}(Ego, t_i) \leftrightarrow ((\textsc{d}(Alter, t_{i-1}) \rightarrow \mathbf{O}\textsc{d}(Ego, t_i)) \wedge (\textsc{c}(Alter, t_{i-1}) \rightarrow \mathbf{O}\textsc{c}(Ego, t_i)))$$

This is very simple, partly because it is first-order rule. Ego's action exclusively depends on Alter's action in the previous round. But the all other action rules are higher-order, which has an important consequence: these action rules depend on the actions of both players in the previous round. In order to compare our formulas with each other we have to convert the TFT's formula into new—redundant—form

$$\begin{aligned}\textsc{tft}(Ego, t_i) \leftrightarrow & ((\textsc{c}(Ego, t_{i-1}) \rightarrow (\textsc{d}(Alter, t_{i-1}) \\ & \rightarrow \mathbf{O}\textsc{d}(Ego, t_i)) \wedge (\textsc{c}(Alter, t_{i-1}) \rightarrow \mathbf{O}\textsc{c}(Ego, t_i))) \wedge (\textsc{d}(Ego, t_{i-1}) \\ & \rightarrow (\textsc{d}(Alter, t_{i-1}) \rightarrow \mathbf{O}\textsc{d}(Ego, t_i)) \wedge (\textsc{c}(Alter, t_{i-1}) \rightarrow \mathbf{O}\textsc{c}(Ego, t_i))))\end{aligned}$$

(ii) Sugden (1986) has proposed a modified version of Tit-for-tat. He referred to it as T_1. Later it has been called *Standing* strategy in the field of indirect reciprocity, but Boyd labeled Sugden's strategy to *Contrite Tit-for-tat* (CTFT) action rule (Boyd, 1989; Panchanathan and Boyd, 2003). Others called this rule as Firm-But-Fair (FBF) strategy (Frean, 1994; Hauert and Schuster, 1998). Although the ‘firm, but fair’ is a familiar, everyday life expression, I prefer the usage of the contrition-related term (Boerlijst et al., 1997). This rule can be characterized by a typical contrite attitude: if the player defected in the previous round, then the strategy prescribes unconditional cooperation. The fault must be corrected. The formula of this strategy is

$$\begin{aligned}\textsc{ctft}(Ego, t_i) \leftrightarrow & ((\textsc{c}(Ego, t_{i-1}) \rightarrow (\textsc{d}(Alter, t_{i-1}) \\ & \rightarrow \mathbf{O}\textsc{d}(Ego, t_i)) \wedge (\textsc{c}(Alter, t_{i-1}) \rightarrow \mathbf{O}\textsc{c}(Ego, t_i))) \wedge (\textsc{d}(Ego, t_{i-1}) \\ & \rightarrow \mathbf{O}\textsc{c}(Ego, t_i)))\end{aligned}$$

This is the most cooperative strategy, it subscribes cooperation in three cases (from the four possibilities). Probably all stories about contrition can be related to CTFT. One of the parables of Jesus describes the same rule: “He that is without sin among you, let him first cast a stone at her.” (John 8.1-11).

(iii) Independent from the theories of reciprocity Kraines and Kraines (1989, 1993) introduced and analyzed the PAVLOV strategy. Rapoport called it *Simpleton* rule (Ridley, 1997), some authors use the *Perfect TFT* name for it (Imhofa et al., 2007), and with the emergence of game theoretic modeling a new term, *Win-Stay-Lose-Shift* (WSLS) appeared on the scene. It is a kind of learning strategy and maybe the most successful rule that outperforms *Simple TFT* in a noisy environment where social error exists