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Numerical simulation of dual-phase-lag bioheat transfer model during thermal therapy

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ABSTRACT

This paper theoretically investigates the thermal behavior in a living biological tissue under various coordinate systems and different non-Fourier boundary conditions with the dual-phase-lag bioheat transfer model during thermal therapy. The properties of Legendre wavelets together with the finite difference scheme are used to find an approximate analytical solution of the present problem. It has been observed that surrounding healthy tissues are less affected in second and third kind of boundary condition when applied along with spherical symmetric coordinate system. Also greater temperature rise and fast achievement of peak hyperthermia temperature is achieved when second and third kind of boundary conditions are used in combination with Cartesian coordinate system. It is observed that due to the presence of blood perfusion and temperature dependent metabolic heat generation term, the dual-phase-lag bioheat transfer model reduces to Pennes bioheat transfer model only when $\tau_q = \tau_T = 0s$, not for arbitrary $\tau_q = \tau_T$. Further, in case of dual-phase-lag bioheat transfer model wave-like or diffusion-like behavior will dominate depends whether the ratio $\tau_q/\tau_T > 1$ or $\tau_q/\tau_T < 1$. Effect of temperature dependent metabolic heat generation rate, thermal conductivity and blood perfusion rate on dimensionless temperature are discussed in details. The whole analysis is presented in dimensionless form.

1. Introduction

Thermal therapy, encompasses all therapeutic treatments of malignant diseases (cancerous cell) based on transfer of thermal energy into or out of the body by various ways (Electromagnetic radiation, Ultrasound, Radio-frequency, Microwaves, Infrared-radiation etc.). It is implemented as a minimally invasive alternative to traditional surgery in the treatment of cancerous cell and benign diseases. In clinical setting, the major objective of thermal therapy is to obtain an effective treatment of cancerous cell without damaging surrounding healthy tissue. Depending on the degree of temperature rise and time to apply them, thermal therapy is classified by Habash et al. [1] into cryoablation ($T \leq -50\,^{o}$C for time $>$ 10 min), long term low-temperature hyperthermia ($40\,^{o}\text{C} \leq T \leq 41\,^{o}$C for time 6–72 h.), moderate-temperature hyperthermia ($41\,^{o}\text{C} \leq T \leq 46\,^{o}$C for time 15–60 min) and high-temperature or thermal ablation ($T \geq 50\,^{o}$C for time 4–6 min). In hyperthermia treatment, the increased temperature at the site of cancerous cell results in changed physiology of diseased cell, which leads to necrosis or apoptosis. The extent of initial tissue necrosis depends on thermal energy applied to the tissue. Necrosis is marked by a passive pathological cell damage followed by an inflammatory response originating from the surrounding tissues whereas, apoptosis represents a genetically controlled cell death. Hyperthermia is an adjuvant therapy means it is applied along with an already established therapy such as chemotherapy or radiotherapy. Therefore, hyperthermia improves the result of the conventional treatment within the framework of multi-modal treatment concepts. For the success of hyperthermia treatment, precise prediction and control of temperature are always needed [2–6]. Heat transfer analysis in living biological tissues is complex due to their non-homogeneous inner structure. It involves heat conduction in solid tissue matrix and blood vessels, convection between blood and tissue, perfusion through capillary tubes within the tissues, metabolic heat generation and evaporation etc. Several bioheat transfer models have been developed in order to model this complex process [7–11].

But due to simplicity, Pennes bioheat transfer (PBHT) model is used most commonly for fast prediction of transient temperature profiles and interpretation of thermal data. The conduction term in the PBHT model is based on macroscopic heat diffusion theory as stipulated by classical Fourier's law:

$$q(r,t) = -k\nabla T(r,t). \tag{1}$$

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Nomenclature

a	antenna constant, m^{-1}
c	specific heat of tissue, J $kg^{-1}K^{-1}$
c_b	specific heat of blood, J $kg^{-1}K^{-1}$
H	heat transfer coefficient, W $m^{-2}K^{-1}$
k	thermal conductivity of tissue, W $m^{-1}K^{-1}$
L	length of tissue, m
P	antenna power, W
q	heat flux, Wm^{-2}
Q_b	heat source due to blood perfusion, W m^{-3}
Q_m	heat source due to metabolic heat generation, W m^{-3}
Q_{mo}	basal metabolic heat generation rate, W m^{-3}
Q_r	heat source due to absorbed electro-magnetic radiation, Wm^{-3}
r	spatial coordinate of tissue, m
$\bar{r}$	distance of the tissue from the skin surface, m
r_p	radius of the treatment probe, m
t	time, s
T	temperature of tissue, °C
T_b	arterial blood temperature, °C
T_f	fluid temperature, °C
T_o	initial temperature, °C
T_w	temperature of the vessel wall, °C
V_{cv}	thermal wave propagation speed, m s^{-1}

Greek symbols

Γ	the number to classify coordinates ($\Gamma = 0, 1, 2$)
ρ	density of tissue, kg m^{-3}
ω_b	perfusion rate of blood, kg $m^{-3}s^{-1}$
τ_q	phase lag of heat flux, s
τ_T	phase lag of temperature gradient, s

Dimensionless variable and similarity criteria

x	dimensionless space coordinate
B_i	Biot number
K_i	Kirchhoff number
F_o	Fourier number or dimensionless time
F_{oq}	dimensionless phase lag of heat flux
F_{oT}	dimensionless phase lag of temperature gradient
P_f	dimensionless blood perfusion coefficient
P_{mo}	dimensionless metabolic heat source coefficient
P_{ro}	dimensionless heat source due to electro-magnetic radiation
θ	dimensionless local tissue temperature
θ_b	dimensionless arterial blood temperature
θ_f	dimensionless fluid temperature
θ_w	dimensionless wall temperature of the tissue
a^*, x^*	dimensionless constants
B	dimensionless constant

It assumes that heat flux vector $q(r, t)$ and temperature gradient $\nabla T(r, t)$ appear at the same instant of time i.e. thermal signal propagates with infinite speed. It means that any thermal disturbance produced at a certain instant of time will be felt throughout the medium at the same instant of time. As heat conduction in a living biological tissue is due to interaction between solid tissue matrix and blood vessels, thermal signal always found to propagates with a finite speed [12]. To solve the paradox occurred in Fourier's law, Cattaneo [13] and Vernotee [14] independently proposed single-phase-lag (SPL) constitutive relation:

$$q(r, t + \tau_q) = -k\nabla T(r, t), \tag{2}$$

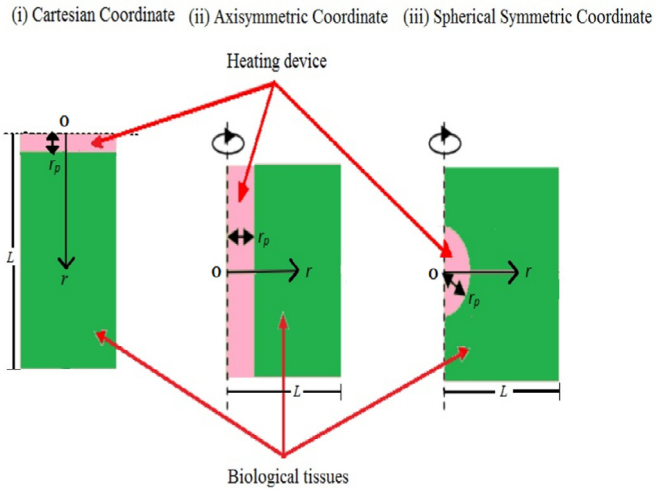

Fig. 1. Models for bioheat transfer analysis at different coordinates.

where a relaxation time τ_q has added to capture the micro-scale responses in time. SPL constitutive relation characterizes wave-like behavior of heat conduction and predicts finite speed for thermal signal

$$V_{cv} = \left(\frac{k}{\rho c \tau_q}\right)^{1/2}. \tag{3}$$

SPL constitutive relation when combined with energy equation gives thermal wave bioheat transfer(TWBHT) model. Although TWBHT model taken into account of micro-scale responses in time, it does not capture micro-scale responses in space. In order to consider the micro-scale responses in both time and space, a phase lag for temperature gradient (τ_T) has introduced in SPL constitutive relation by Tzou [15,16]

$$q(r, t + \tau_q) = -k\nabla T(r, t + \tau_T). \tag{4}$$

According to this relation, the temperature gradient at a point r at time $t + \tau_T$ corresponds to the heat flux at r at time $t + \tau_q$. The corresponding model is called dual-phase-lag bioheat transfer(DPLBHT) model. A kind of generalization of DPLBHT model has been done by Zhang [17] based on the theory of porous media and non-equilibrium heat transfer in biological tissues. In this theory, phase lag times has been expressed in terms of porosity, coupling factor, heat capacities and thermal conductivities of blood and tissues. Vadasz [18] shows the lack of local thermal equilibrium in DPL heat conduction for porous media and demonstrated that the condition required for oscillatory solutions are not physically attainable.

The coordinate system and boundary condition used for analysis of thermal data changes according to the treatment method. Fig. 1 [19] shows the geometry of different kinds of coordinates considered in this study. In Fig. 1, r_p denotes the radius of the treatment probe. The Cartesian coordinate ($\Gamma = 0$) corresponds to surface heating or cooling. Haugk et al. [20] used body surface cooling with a cooling pad and approximated it by Cartesian coordinates. Axisymmetric coordinate ($\Gamma = 1$) is suitable for treatment using a heating or cooling probe whereas spherical symmetric coordinate ($\Gamma = 2$) can be better approximated when the heating or cooling section is small. Radio-frequency ablation uses a heating probe and can be approximated by axisymmetric or spherical symmetric coordinates as the study by Haugk et al. [20] suggests. Cheng and Liu [21], Kengne and Lakhssasi [22] numerically studied heat transport phenomenon in biological tissues using spherical coordinates. Akbarzadeh and Chen [23] has derived heat conduction equations based on DPL theory and