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Bioheat transfer problem for one-dimensional spherical biological tissues

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ABSTRACT

Based on the Pennes bioheat transfer equation with constant blood perfusion, we set up a simplified one-dimensional bioheat transfer model of the spherical living biological tissues for application in bioheat transfer problems. Using the method of separation of variables, we present in a simple way the analytical solution of the problem. The obtained exact solution is used to investigate the effects of tissue properties, the cooling medium temperature, and the point-heating on the temperature distribution in living bodies. The obtained analytical solution can be useful for investigating thermal behavior research of biological system, thermal parameter measurements, temperature field reconstruction and clinical treatment.

1. Introduction

Spatiotemporal temperature distribution in living biological tissues plays a vital role in many physiological processes. Investigation of bioheat transfer problems requires the evaluation of temporal and spatial distributions of temperature. This class of problems has been traditionally addressed using the Pennes bioheat equation [1]. Scientific research in the bioheat transfer research field has paved a key foundation in hyperthermia cancer therapy, thermal diagnosis, cryogenic surgery etc. [2–8]. The quantitative, qualitative, and accurate analysis of bioheat transfer is to effectively understand and model the heat transfer mechanism of the biological system.

Heat transfer analysis on thermal medical problems, such as the thermal diagnostics [9] and thermal comfort analysis [10,11], thermal parameter estimation [12–16], or burn injury evaluation [17], usually has to simultaneously face the transient or spatial heating both on skin surface and in interior of the biological bodies. The complexity underlying in this class of medical problems remains not only for its heterogeneity and anisotropy but also for conduction, convection, and radiation heat flow, cell's metabolism, and blood perfusion etc. Therefore, to obtain a flexible solution, which is capable of solving any one of the above thermal medical problems, is very desirable. Indeed, analytical solutions reflect actual physical feature of the models and can be used as standards to verify the corresponding numerical results and as a proof to the reasonability of in-vitro mode analysis. Although people relied too much on numerical approaches such as finite difference method (FDM), finite element method (FEM), and boundary element method (BEM) for solving thermal medical problems, the analytical solutions, if they can be obtained, are often preferred [18,19]. Analytical solutions are very attractive since their efficiency depends weakly on the dimensions of the problem, in contrast to the numerical methods. Knowing analytical solution, temperature at a desired point at a given time can be performed independently from that of the other points within the domain, which can be an asset when temperatures are needed at only some isolated sites or times. It is also important to point out that analytical solutions of bioheat transfer problems will save computational time greatly, which is valuable in some hyperthermia practices. Motivated by the importance of exact solutions of bioheat transfer problems in hyperthermia cancer therapy, thermal diagnosis, or cryogenic surgery, we aimed in this paper to present analytical solutions to the Pennes' bioheat model in one-dimensional spherical coordinate system with relatively complex boundary or volumetric heating conditions [20].

Various techniques have been performed to obtain exact solutions of bioheat transfer problems in one dimensional Cartesian coordinate [8,21–27]. In our knowledge, a combination of the method of separation of variables with the Green's function method has not yet been applied for explicitly solving bioheat transfer problems for spherical symmetry. In this paper, we combine the Fourier method (method of separation of variables) with the Green's function method to derive the exact solution of one-dimensional model of the spherical living tissue. Such a combination of the two methods has been used by Durkee and Antich [28,29] when addressing the time-dependent Pennes' equation in 1-D multi-region Cartesian and spherical

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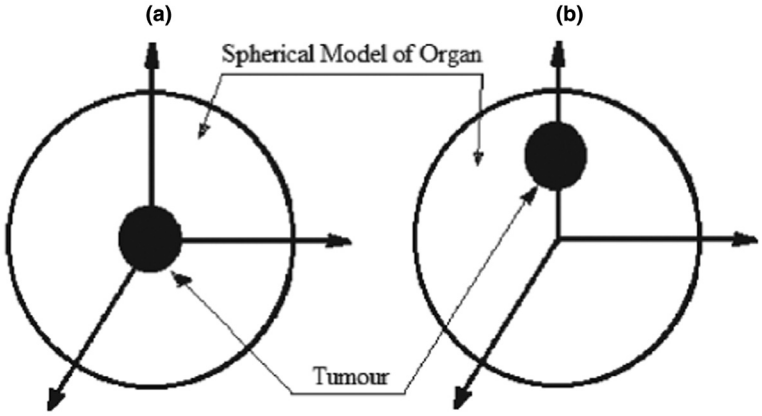

Fig. 1. Spherical model of organ showing the tumour either at the center of the model (a) or far from the center of the model (b).

geometry. In most of the existing analytical studies of bioheat transfer in biological tissues based on either the method of separation of variables, or the Green's function method, or a combination of these two methods, the solutions to the bioheat transfer problem are either for a steady state, heat conduction equations, an infinite domain, or for a constant heating at skin surface or inside the tissue volume, which may not be practical for some real bio-thermal situations (see for example Refs. [25,26,30]). The main aim of combining in the present work the method of separation of variables with the Green's function method is to find analytical exact solution that may incorporate relatively complete situations such as the finite tissue domain, the transient or space-dependent boundary conditions and volumetric heating. The rest of the paper is organized as follows. In Section 2, we describe the model under consideration and present its analytical solution. The application of obtained solution to the tissue's internal temperature distribution and the thermal effect of tumors are discussed in Section 3, and the main results are summarized in Section 4.

2. Model and solution

An important number of theoretical analysis on bioheat transfer problems is based on the Pennes equation [1], which describes the influence of blood flow on the temperature distribution in the tissue in terms of volumetrically distributed heat sinks or sources. Thus it will also be used in this paper. More precisely, we consider in this paper one-dimensional (1-D) case of Pennes bioheat model with constant thermal parameters, which is written in 1-D spherical coordinate system as

$$\rho c\frac{\partial T}{\partial t} = k\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \omega_b\rho_b c_b(T_a - T) + q_m + Q(r,t), \qquad (1)$$

where ρ, c, and k are the density, the specific heat, and the thermal conductivity of the tissue, respectively, ρ_b and c_b denote density and specific heat of blood, ω_b stands for the blood perfusion, T_a and T are the arterial temperatures which are treated as a constant and the tissue temperature at a given position for a given time, respectively, q_m is the metabolic heat generation, and Q is the heat source due to spatial heating. In Eq. (1), $r^2 = x^2 + y^2 + z^2$, where (x, y, z) are the rectangular coordinates of a point of the tissue, situated at a distance r from the center of the model; here, $0 \le x \le L$, where L is the distance from the skin surface ($x = L$) to the body core ($x = 0$).

Following Hossain and Mohammadi [31], the initial temperature field for the basal state of biological bodies in the case of axially symmetrical model can be obtained through solving the following problem:

$$\begin{cases} \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2\dfrac{dT_0}{dr}\right) - \alpha T_0 + \beta = 0 \\ \left.\dfrac{dT_0}{dr}\right|_{r=0} = 0, \\ \left.-k\dfrac{dT_0}{dr}\right|_{r=R_1} = h_a(T_0 - T_e)|_{r=R_1}, \end{cases} \qquad (2)$$

where $\alpha = \omega_b\rho_b c_b/k$, $\beta = (\omega_b\rho_b c_b T_a + q_m)/k$, and $T_0(r) = T(r,0)$ is steady-state temperature fields prior to heating, R_1 is the radius of the concerned tissue, h_a is the heat exchange coefficient which accounts for both the convection and radiation heat loss on the tissue surface, and T_e is the surrounding air temperature. Here, the skin surface is defined at $r = R_1$ while the body core at $r = 0$. We assume that the tumour is located either at the center of the spherical model (see Fig. 1(a)) or far from the center of the spherical model (see Fig. 1(b)).

The solution to problem (2) is

$$T_0(r) = \frac{\beta}{\alpha} + \frac{2R_1^2 h_a(\alpha T_e - \beta)\sinh\left[\sqrt{\alpha}r\right]}{\alpha D r} \qquad (3)$$

where

$$D = \left(R_1 k\sqrt{\alpha} + k - h_a R_1\right)\exp\left[-\sqrt{\alpha}R_1\right] + \left(h_a R_1 + R_1 k\sqrt{\alpha} - k\right)\exp\left[\sqrt{\alpha}R_1\right].$$

During the practical thermal processes, the boundary condition (BC) at the skin surface is often time-dependent. For axially symmetrical model the boundary conditions at tumor surface and at the skin surface are described respectively as

$$\left.-k\frac{\partial T}{\partial r}\right|_{r=0} = 0, \qquad (4a)$$

$$\left.-k\frac{\partial T}{\partial r}\right|_{r=R_1} = h_f\left[T - f(t)\right]\Big|_{r=R_1}, \qquad (4b)$$

where $f(t)$ is the time-dependent temperature of the cooling medium, h_f is the transfer coefficient due to convection and radiation (h_f is the heat transfer coefficient in the direction of heat flow on the boundary skin surface), and $\partial T/\partial r$ is the partial derivative of T along the outward direction normal to the skin surface. No heat loss is assumed in remaining regions. The flux condition (4b) describes the exchange between body and surrounding medium. The choice of BC (4b) is motivated by the fact that typical boundary conditions at the skin surface for cancer hyperthermia or thermal comfort analysis are usually the