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Optimal escapement in stage-structured fisheries with environmental stochasticity

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ABSTRACT

Stage-structured population models are commonly used to understand fish population dynamics and additionally for stock assessment. Unfortunately, there is little theory on the optimal harvest of stage-structured populations, especially in the presence of stochastic fluctuations. In this paper, we find closed form optimal equilibrium escapement policies for a three-dimensional, discrete-time, stage-structured population model with linear growth, post-harvest nonlinear recruitment, and stage-specific pricing and extend the analytic results to structured populations with environmental stochasticity. When only fishing reproductive adults, stochasticity does not affect optimal escapement policies. However, when harvesting immature fish, the addition of stochasticity can increase or decrease optimal escapement depending on the second and third derivative of the recruitment function. For logistic recruitment, stochasticity reduces optimal immature escapement by a multiplicative factor of one over one plus the variance of the environmental noise. Using hard clam, *Mercenaria mercenaria*, as an example and assuming Beverton–Holt recruitment, we show that optimal fishing of hard clam targets the immature stage class exclusively and that environmental stochasticity increases optimal escapement for low discount rates and decreases optimal escapement for high discount rates.

1. Introduction

Fisheries biologists, managers and economists widely recognize that traditional one-dimensional bioeconomic models are too simple for developing management guidelines for the majority of real-life fisheries [1], as policies derived from such models can drastically reduce profits and stock sizes when naively applied to age-structured populations [2]. While determining optimal age-specific harvest policies is a classic problem in bioeconomics [3–8], and an active area of both theoretical and applied research [9–16], the effect of stage structure and environmental stochasticity on optimal fisheries management is poorly understood.

Stage-structured models are often used to understand fish population dynamics and perform stock assessment [17,18]. It is usually more convenient for managers to obtain data on fish size or life stage rather than age. Techniques for aging organisms can be expensive and time consuming and in extreme cases logistically infeasible [19]. In addition, fish prices are often based on discrete size classes or life stages [20,21]. While developing optimal harvest rules for size and stage-structured populations can potentially improve fisheries management, it is more challenging to solve for optimal strategies in this framework due to the lack of sparsity in the equations for stock dynamics.

Past studies have simplified the problem by limiting transitions between stage classes [22,23] or using continuous time models, including two-dimensional ordinary differential equations (e.g. [24]) and partial differential equations [25,26]. However, fisheries biologists usually use discrete-time models for stock assessment [27] and variability among individuals within a population can lead to a wide range of stage transitions, previously unexplored with respect to optimization.

Tahvonen's model [23] is the most similar to our deterministic setup, but differs in a few key ways. We assume that harvest occurs prior to growth and recruitment, as is usually the case for migrating fish populations such as eel and salmon [28,29] and that individuals can skip stages. However, the biggest difference between our two approaches is that we consider the addition of environmental stochasticity.

While the effect of environmental stochasticity on optimal harvest has been widely studied for one-dimensional bioeconomic models (e.g. [30]), little is known about how stochasticity affects optimal harvest in structured populations. Of the few studies that exist, stochasticity is typically only included in the form of random recruitment,

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and usually independent of spawning biomass [12,22]. In addition, solutions heavily rely on numerical simulation and error-bound approximation [22,31].

In this paper, to our knowledge, we provide the first analytic optimal stationary escapement solution for a demographically structured population model with endogenous, nonlinear recruitment and environmental stochasticity affecting all classes. We find that with the addition of environmental stochasticity, the optimal escapement of reproductive adults remains unchanged from the deterministic case, if harvest occurs prior to recruitment. However, in the case of immature harvest, fishing should either be more aggressive or conservative than the deterministic case depending on the second and third derivatives of the recruitment function. For example, if the recruitment function is logistic, escapement should decrease by a factor of one over one plus the variance of the environmental noise. We use historical data from New York state's hard clam fishery to provide a concrete example of our theoretical results.

2. The deterministic model

Consider a harvested stage-structured fish population, where events occur in the following order: (1) census (2) harvest, (3) reproduction, and (4) natural mortality and growth. That is,

$$B_{1,t+1} = R(B_{3,t} - h_t) + a_{11}B_{1,t} \tag{1}$$

$$B_{2,t+1} = a_{21}B_{1,t} + a_{22}(B_{2,t} - \eta_t) \tag{2}$$

$$B_{3,t+1} = a_{31}B_{1,t} + a_{32}(B_{2,t} - \eta_t) + a_{33}(B_{3,t} - h_t), \tag{3}$$

where $B_{1,\,t}$, $B_{2,\,t}$, and $B_{3,\,t}$ are the biomass of juvenile (stage 1), immature (stage 2) and adult (stage 3) fish at time t, respectively. The juvenile stage consists of fish too small to reproduce or catch. The immature stage consists of all fish large enough to catch, but still cannot reproduce. At time t, h_t and η_t units of biomass are harvested from the adult and immature fish population and we define $\sigma_t \equiv B_{3,t} - h_t$ and $s_t \equiv B_{2,t} - \eta_t$ as the corresponding amount of adult and immature biomass that escape harvest. The remaining fish survive and grow, where a_{ij} is the per unit biomass contribution, from the biomass that escaped harvest in stage j, at time t, to the biomass in stage i, at time $t+1$.

We assume, $0 \le a_{ij} \le m_i/m_j$, for all $i > j$, where m_i is the average mass of a stage i individual and that $a_{ij} < 1$ for all $i = j$. This means that the population's biomass can only increase through reproduction and transitioning between different stages. If a_{ii} were greater than one for some i, then, in the absence of harvest, an initial amount of biomass in the ith stage would grow to infinity, even if there was no reproduction.

Adults that escape harvest, reproduce, generating offspring with total biomass described by a bounded, positive, smooth, concave, density-dependent recruitment function $R(\sigma_t)$, where σ_t is the spawning biomass at time t and $R(0) = 0$. We can write the model in matrix notation as

$$\vec{B}_{t+1} = A(\vec{B}_t - \vec{h}_t) + \vec{R}_t, \tag{4}$$

where

$$\vec{B}_t = \begin{pmatrix} B_{1,t} \\ B_{2,t} \\ B_{3,t} \end{pmatrix}, A = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

$$\vec{h} = \begin{pmatrix} 0 \\ \eta_t \\ h_t \end{pmatrix}, \vec{R}_t = \begin{pmatrix} R(B_{3,t} - h_t) \\ 0 \\ 0 \end{pmatrix}.$$

$B_{1,\,t}$ is bounded for all time, because R is bounded and $a_{11} < 1$. If follows that $B_{2,\,t}$ and $B_{3,\,t}$ are bounded since $a_{ij} < 1$ for $i = j$.

Note, the model unconventionally tracks biomass rather than population abundance. In classic stage-structured models of abundance, a_{ij} is the probability of an individual surviving and transitioning from stage j to stage i. However, in our model, a_{ij} is a composite parameter which additionally includes growth. Because of our assumptions on R and a_{ij}, a simple rescaling, from biomass to abundance, of any admissible parameterization of (4) yields a valid parameterization of the classic stage-structured model presented in [32, Chs. 3, 4 and 16]. For example, if $\hat{a}_{ij}$ is an entry in the classic transition matrix model, then $\hat{a}_{ij} = a_{ij}m_j/m_i$, where m_i is the average mass of a stage i individual.

We wish to maximize total discounted revenue, where revenue is a linear function of harvest,

$$\max_{h_t,\eta_t} \left\{ \sum_{t=0}^{\infty} \rho^t (p_3 h_t + p_2 \eta_t) \right\}, \tag{5}$$

with p_2 and p_3, the price per unit biomass of immature and adult fish, respectively, and $\rho = 1/(1+\delta)$, the discrete discount factor, with discount rate $\delta \ge 0$. Harvest is also subject to the constraints $0 \le h_t \le B_{3,\,t}$ and $0 \le \eta_t \le B_{2,\,t}$.

3. Analysis of the deterministic model

3.1. Optimal equilibrium escapement

We use the Karush–Kuhn–Tucker theorem (p. 61 of [1]) to solve for the equilibrium optimal harvest policy. The Lagrangian for the problem is

$$\begin{aligned}\mathscr{L} = \sum_{t=0}^{\infty} \rho^t \{ & p_3 h_t + p_2 \eta_t + \rho\lambda_{1,t+1}[R(B_{3,t} - h_t) + a_{11}B_{1,t} - B_{1,t+1}] \\ & + \rho\lambda_{2,t+1}[a_{21}B_{1,t} + a_{22}(B_{2,t} - \eta_t) - B_{2,t+1}] \\ & + \rho\lambda_{3,t+1}[a_{31}B_{1,t} + a_{32}(B_{2,t} - \eta_t) + a_{33}(B_{3,t} - h_t) - B_{3,t+1}] \\ & + \mu_{1,t}h_t + \mu_{2,t}[B_{3,t} - h_t] + \mu_{3,t}\eta_t + \mu_{4,t}[B_{2,t} - \eta_t]\}\end{aligned} \tag{6}$$

where $\rho\lambda_{i,t+1}$ is the current value shadow price for $B_{i,\,t}$, the money someone would pay for a small additional amount of biomass in the ith stage at time $t+1$ and $\mu_{i,\,t}$ are the multipliers for the inequality constraints on harvest. The Karush–Kuhn–Tucker necessary conditions for and optimal solution are

$$\rho a_{11}\lambda_{1,t+1} + \rho a_{21}\lambda_{2,t+1} + \rho a_{31}\lambda_{3,t+1} - \lambda_{1,t} = 0, \tag{7}$$

$$\rho a_{22}\lambda_{2,t+1} + \rho a_{32}\lambda_{3,t+1} - \lambda_{2,t} + \mu_{4,t} = 0, \tag{8}$$

$$\rho a_{33}\lambda_{3,t+1} + \rho R'(B_{3,t} - h_t)\lambda_{1,t+1} - \lambda_{3,t} + \mu_{2,t} = 0, \tag{9}$$

$$p_2 - \rho a_{22}\lambda_{2,t+1} - \rho a_{32}\lambda_{3,t+1} + \mu_{3,t} - \mu_{4,t} = 0, \tag{10}$$

$$p_3 - \rho R'(B_{3,t} - h_t)\lambda_{1,t+1} - \rho a_{33}\lambda_{3,t+1} + \mu_{1,t} - \mu_{2,t} = 0, \tag{11}$$

$$\mu_{1,t}h_t = 0 \text{ and } \mu_{2,t}[B_{3,t} - h_t] = 0, \tag{12}$$

$$\mu_{3,t}\eta_t = 0 \text{ and } \mu_{4,t}[B_{2,t} - \eta_t] = 0, \tag{13}$$

$$\text{and } \mu_{i,t} \ge 0 \text{ for all } i \in \{1, 2, 3, 4\}. \tag{14}$$

At steady state, the above system can be solved analytically, yielding optimal equilibrium escapement rules for immature and adult biomass, which we define as s^* and σ^*, respectively. We use the star superscript $(^*)$ and the omission of the time subscript to denote steady state values for each variable under the optimal equilibrium