Mathematical Biosciences 269 (2015) 86–93

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

Analysis of a waterborne disease model with socioeconomic classes

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ARTICLE INFO

Article history:
Received 22 December 2014
Revised 29 July 2015
Accepted 28 August 2015
Available online 7 September 2015

Keywords:
Cholera
Socioeconomic status
Basic reproduction number
Heterogeneity
Disease dynamics

ABSTRACT

Waterborne diseases such as cholera continue to pose serious public health problems in the world today. Transmission parameters can vary greatly with socioeconomic class (SEC) and the availability of clean water. We formulate a multi-patch waterborne disease model such that each patch represents a particular SEC with its own water source, allowing individuals to move between SECs. For a 2-SEC model, we investigate the conditions under which each SEC is responsible for driving a cholera outbreak. We determine the effect of SECs on disease transmission dynamics by comparing the basic reproduction number of the 2-SEC model to that of a homogeneous model that does not take SECs into account. We conclude by extending several results of the 2-SEC model to an n-SEC model.

1. Introduction

Waterborne diseases such as cholera remain a major public health problem in developing countries, where outbreaks continue to occur and are intensely interconnected with poverty, malnutrition and poor sanitation [1,2]. Approximately 700,000 children die due to dehydration every year from diarrhea caused by unsafe water [3]. According to the World Health Organization [4], cholera incidence has increased steadily since 2005. In 2011, 58 nations reported 589,854 cholera cases and 7728 deaths [5,6]. Due to poor surveillance and under-reporting, the above statistics are likely to be much lower than the actual cases and deaths globally [7].

The position of an individual or group within a hierarchical social structure depends on occupation, education and income. This position is referred to as socioeconomic status [8]. Individuals in a low socioeconomic class (SEC) are characterized by poverty, malnutrition, poor sanitation and low standards of living whereas individuals in a high SEC are known for high standards of living, quality education, good jobs with higher income, clean living environments and ready access to clean water. Apart from impoverished countries, low SEC individuals can also be found in places such as refugee camps, areas devastated by war, famine or natural disasters. While everyone is susceptible to waterborne diseases, individuals in low SECs are likely to be more vulnerable to these diseases [1,9] due to the lack of access to a clean water source.

Most societies or communities are made up of individuals from multiple SECs. We can study the dynamics of a waterborne disease in these communities by formulating and analyzing an appropriate mathematical model. Such a model for a waterborne disease incorporating SECs is expected to improve our understanding of the dynamics of the disease and inform public health interventions.

A number of mathematical models have explored the dynamics of infectious disease, including waterborne diseases [1,10–21]. These studies have contributed immensely to our understanding of the dynamics of these diseases as well as possible control strategies. Ngonghala [22] explored combining economic growth models with models for infectious disease. The impact of economic classes on disease transmission was studied by Pedro and Tchuenche [23] in a mathematical model for the transmission HIV/AIDS. In their analysis, they considered two social classes, namely the poor and the rich, including transmission from one class to another. The dynamics of waterborne disease differ greatly from those of HIV, and the inclusion of economic status in models has not yet been thoroughly explored. The aim of this study is to improve our understanding of the effect of heterogeneity in transmission due to socioeconomic status on the dynamics of waterborne disease by formulating an appropriate mathematical epidemiological model that incorporates multiple SECs.

We note the movement of individuals between different SECs often occurs over a timescale on the order of years, slower than the timescale of the outbreak [9]. However, in developing countries, this is not usually the case. For example, in South Africa, most universities are situated in areas of high socioeconomic class. Thus, once a student graduates from secondary school in a rural area (low SEC), s/he moves immediately into an environment with a higher socioeconomic class. Further, in countries which seek to rapidly address the

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http://dx.doi.org/10.1016/j.mbs.2015.08.016
0025-5564/

imbalances of the past (In South Africa this has to do with ensuring greater representation of the majority in various sectors of the population; in India, this has to do with increasing the representation of the different castes (especially the Dalits)) changes occur on a very short time scale. Specifically, South Africa has introduced a form of economic empowerment known as Broad-Based Black Economic Empowerment (BBBEE) [24]. This Act serves to provide Black South Africans with a boost in their economic status through a variety of mechanisms. As a result of this intervention, the socioeconomic status of many Black South Africans has improved dramatically, often in the timeframe of a few weeks. Given these circumstances, the change in socioeconomic status can indeed occur on the same timescale as that of the disease.

We present an n-SEC model in Section 2. Next, we consider a special case of the model when $n = 2$ in Section 3. In Section 4, some of the results of the 2-SEC model are extended to the general n-SEC model.

2. Model formulation

Let N be the total human population of a community at risk for waterborne disease infections. We partition N into n socioeconomic classes (SECs) or homogeneous subpopulations of size N_j, each with their own water source (Fig. 1). Each SEC is made up of susceptible S_j, infected I_j and recovered R_j individuals together with a compartment W_j that measures pathogen concentration in the water reservoir. John Snow demonstrated that the waterborne disease cholera was spread through contact with a contaminated drinking pump during an outbreak in London (indirect transmission) [25]. Direct (or person to person) transmission has also been suggested to contribute to the spread of waterborne disease [26], though the relative importance of direct and indirect transmission is likely to vary greatly among outbreaks [19,27]. Here we assume no person to person transmission and consider only indirect transmission through contact with contaminated water, as it is often considered to be the main driver of waterborne disease outbreaks [20,21]. As a result, secondary infections are generated when an infected individual sheds pathogens into the water source to which susceptible individuals subsequently come into contact. Susceptible individuals S_j become infected through contact with the contaminated water source W_j at rate b_j. Infected individuals I_j can contaminate the water source by shedding pathogens at rate ν_j and recover naturally at rate γ_j. Pathogens grow naturally at rate α and decay at rate ξ. Natural death occurs in SEC i at rate μ_i.

Lower SECs have fewer resources to treat water, or to prevent shedding (such as rainwater washing feces into drinking water) than higher SECs, who have more access to clean water sources along with the ability to treat the water before drinking. Furthermore, recovery rates of the higher SECs will likely be greater than that of the lower SECs due to the ability to reach medical treatment in a timely manner. We therefore assume that SEC 1 is the lowest class followed by SEC 2 in this order until SEC n, for $n > 2$. Based on these, we obtain the inequalities: $b_1 > b_2 > \cdots > b_n$, $\nu_1 > \nu_2 > \cdots > \nu_n$ and $\gamma_1 < \gamma_2 < \cdots < \gamma_n$. Some individuals can improve their SEC as they acquire more education or obtain a better paying job. On the other hand, some individuals may lose their jobs leaving them with lower income. In this case such individuals will move down to a lower SEC. We assume that $S_j(t)$ migrate to $S_k(t)$ at the rate δ_{jk}. In particular, we assume that $S_j(t)$ can migrate to the next lower SEC $S_{j-1}(t)$ or to the next higher SEC $S_{j+1}(t)$. Similarly, we assume that $I_j(t)$ migrate to $I_k(t)$ at the rate l_{jk}. We do not consider change in SEC of $R_j(t)$ since the migration of $R_j(t)$ will not affect the spread of infection. Putting these formulations and assumptions together, we obtain the model

$$
\begin{aligned}
\dot{S}_1(t) &= \mu_1 N_1 - b_1 S_1 W_1 - \mu_1 S_1 - \delta_{12} S_1 + \delta_{21} S_2,\\
\dot{I}_1(t) &= b_1 S_1 W_1 - (\mu_1 + \gamma_1) I_1 - l_{12} I_1 + l_{21} I_2,\\
\dot{W}_1(t) &= \nu_1 I_1 - \sigma W_1,\\
\dot{R}_1(t) &= \gamma_1 I_1 - \mu_1 R_1,\\
\dot{S}_2(t) &= \mu_2 N_2 - b_2 S_2 W_2 - \mu_2 S_2 - \sum_{j=1}^{3} \delta_{2j} S_2 + \sum_{j=1}^{3} \delta_{j2} S_j,\\
\dot{I}_2(t) &= b_2 S_2 W_2 - (\mu_2 + \gamma_2) I_2 - \sum_{j=1}^{3} l_{2j} I_2 + \sum_{j=1}^{3} l_{j2} I_j,\\
\dot{W}_2(t) &= \nu_2 I_2 - \sigma W_2,\\
\dot{R}_2(t) &= \gamma_2 I_2 - \mu_2 R_2,\\
\vdots &= \vdots\\
\dot{S}_n(t) &= \mu_n N_n - b_n S_n(t) W_n - \mu_n S_n(t) - \delta_{n,n-1} S_n + \delta_{n-1,n} S_{n-1},\\
\dot{I}_n(t) &= b_n S_n W_n - (\mu_n + \gamma_n) I_n - l_{n,n-1} I_n + l_{n-1,n} I_{n-1},\\
\dot{W}_n(t) &= \nu_n I_n - \sigma W_n,\\
\dot{R}_n(t) &= \gamma_n I_n - \mu_n R_n.
\end{aligned}
\tag{1}
$$

where $\sigma = \xi - \alpha > 0$, $j = 1, 2, \ldots, n$ and $\delta_{jj} = \delta_{kk} = l_{jj} = l_{kk} = 0\ \forall\ j, k$. By rescaling model (1) as follows: $i_j = I_j/N, s_j = S_j/N, r_j = R_j/N, w_j = \sigma W_j/(\nu_j N)$ and $\beta_j = b_j \nu_j N/\sigma$, we obtain the non-dimensionalized model given by

$$
\begin{aligned}
\dot{s}_1(t) &= \mu_1 n_1 - \beta_1 s_1 w_1 - \mu_1 s_1 - \delta_{12} s_1 + \delta_{21} s_2,\\
\dot{i}_1(t) &= \beta_1 s_1 w_1 - (\mu_1 + \gamma_1) i_1 - l_{12} i_1 + l_{21} i_2,\\
\dot{w}_1(t) &= \sigma (i_1 - w_1),\\
\dot{r}_1(t) &= \gamma_1 i_1 - \mu_1 r_1,\\
\dot{s}_2(t) &= \mu_2 n_2 - \beta_2 s_2 w_2 - \mu_2 s_2 - \sum_{j=1}^{3} \delta_{2j} s_2 + \sum_{j=1}^{3} \delta_{j2} s_j,\\
\dot{i}_2(t) &= \beta_2 s_2 w_2 - (\mu_2 + \gamma_2) i_2 - \sum_{j=1}^{3} l_{2j} i_2 + \sum_{j=1}^{3} l_{j2} i_j,\\
\dot{w}_2(t) &= \sigma (i_2 - w_2),\\
\dot{r}_2(t) &= \gamma_2 i_2 - \mu_2 r_2,
\end{aligned}
\tag{2}
$$

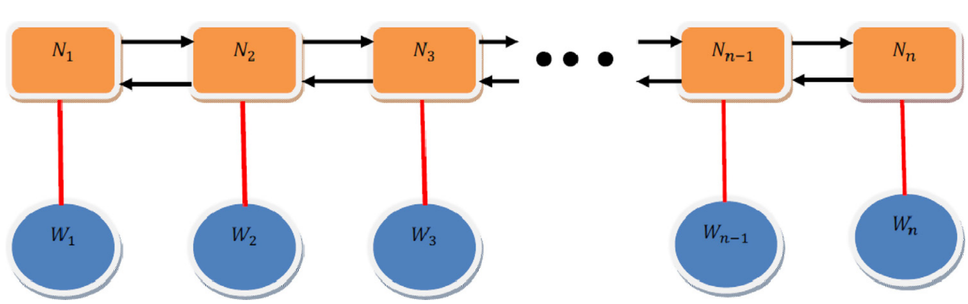

Fig. 1. Schematic representation of n-socioeconomic classes in a population. Each box represents the population in an SEC and each circle below it represents the water source for the corresponding class. The horizontal arrows indicate migration of individuals across socioeconomic classes as their socioeconomic status changes.