Mathematical Biosciences 263 (2015) 143–160

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

Some analytical and numerical approaches to understanding trap counts resulting from pest insect immigration

Daniel Bearup [a], Natalia Petrovskaya [b], Sergei Petrovskii [c,*]

[a] ICBM, University of Oldenburg, Carl-von-Ossietzky-Strasse 9-11, D-26111 Oldenburg, Germany
[b] School of Mathematics, University of Birmingham, Birmingham B15 2TT, UK
[c] Department of Mathematics, University of Leicester, University Road, Leicester LE1 7RH, UK

A R T I C L E   I N F O

Article history:
Received 11 February 2014
Revised 19 February 2015
Accepted 25 February 2015
Available online 2 March 2015

Keywords:
Pest monitoring
Insect trapping
Random walk
Diffusion
Finite differences

A B S T R A C T

Monitoring of pest insects is an important part of the integrated pest management. It aims to provide information about pest insect abundance at a given location. This includes data collection, usually using traps, and their subsequent analysis and/or interpretation. However, interpretation of trap count (number of insects caught over a fixed time) remains a challenging problem. First, an increase in either the population density or insects activity can result in a similar increase in the number of insects trapped (the so called “activity-density” problem). Second, a genuine increase of the local population density can be attributed to qualitatively different ecological mechanisms such as multiplication or immigration. Identification of the true factor causing an increase in trap count is important as different mechanisms require different control strategies. In this paper, we consider a mean-field mathematical model of insect trapping based on the diffusion equation. Although the diffusion equation is a well-studied model, its analytical solution in closed form is actually available only for a few special cases, whilst in a more general case the problem has to be solved numerically. We choose finite differences as the baseline numerical method and show that numerical solution of the problem, especially in the realistic 2D case, is not at all straightforward as it requires a sufficiently accurate approximation of the diffusion fluxes. Once the numerical method is justified and tested, we apply it to the corresponding boundary problem where different types of boundary forcing describe different scenarios of pest insect immigration and reveal the corresponding patterns in the trap count growth.

1. Introduction

Pest insect monitoring is an important component of the integrated pest management (IPM) [6,20,28]. Its purpose is to obtain a reliable estimate of the pest abundance based on data collected in the field. A reliable estimate is required in order to help the IPM specialists to make an informed decision about control measures, e.g. application of chemical pesticides when the pest density exceeds a certain threshold [16,38] and yet to avoid their unjustified use.

Data on insect abundance are usually collected with traps [26,36]. After a trap is set up in the field and has been exposed for a certain time, it catches a certain number n_1 of insects of a given species. This number is called a trap count; if, for instance, the trap was exposed for one day, it is called the daily count. In case $n_1 > 0$, this can be regarded as proof that the species is present in the vicinity of the trap. However, relating the trap count to the population density is a much more difficult problem. Previous approaches tended to provide a relative rather than absolute estimate [40]. Recently, [34,35] showed that, if information is available about the insect movement pattern, the population density can be obtained by placing the sequence of daily counts against the predictions of a relevant mean-field mathematical model of the population dispersal. The simplest model of this type is the diffusion equation, which assumes that insects perform the Brownian motion, and indeed there is considerable evidence that they often do so [41] although this may not always be readily seen from data [19,33].

The diffusion equation is a well-known and well-studied model and, in case of one spatial dimension, its general solution can usually be found analytically, albeit not always in a compact form. The situation is essentially different in case of higher dimension. In a 2D case, analytical solution of the diffusion equation is only possible if the domain possess a certain symmetry, e.g. has the shape of a rectangle or a disk. Even then, however, the analytical solution often becomes impractical. For instance, in a disk-shaped domain, the solution can only be obtained as an infinite series

---

* Corresponding author. Tel.: +44 116 252 3916; fax: +44 116 252 3915.
E-mail address: sp237@le.ac.uk (S. Petrovskii).

http://dx.doi.org/10.1016/j.mbs.2015.02.008
0025-5564/

where the coefficients are solutions of transcendent algebraic equations and hence can only be found numerically. In this situation, i.e. when numerical methods have to be used anyway, a reasonable alternative approach is to solve the diffusion equation numerically right away instead of using the semi-analytical method described above.

In this paper, we use a combination of analytical and numerical approaches to investigate the patterns in the trap count resulting from different boundary conditions as given by different ecological situations. One of the goals of pest insect monitoring is to detect an early sign of population growth in order to prevent a pest outbreak. It is therefore important to understand how an increase in the population density can be reflected by the trap count. It seems intuitively obvious that a larger population size should eventually result in a larger trap count. The rate of increase can, however, be different as an increase in the total population size can be attributed to different reasons. It can result from the growth of the local population, i.e. the population inside the given field, and indeed some insect species are capable of producing several generations in one year. It can also result from migration of the pest into the field from an external source, i.e. from another habitat. For the goals of pest control, it is important to distinguish between these situations (as the control measures are likely to be different) as well as between different immigration patterns. Misidentification of the reason behind the pest abundance increase can result in a completely wrong estimate of the pest population density and that can have a detrimental effect on the efficiency of control measures. Here we are mostly concerned with the effect of immigration as the most common scenario; the effect of local population growth will be considered elsewhere.

There are a variety of numerical methods that can be used to solve numerically the diffusion equation; e.g. see [39]. However, we mention here that that their applicability and efficiency depend on the geometry of the domain. A typical domain in the trapping problem is not simply connected as it has an external boundary (i.e. the boundary of the monitored farm field) and the internal boundary (the boundary of the trap). Moreover, the size of the trap is usually much less than the size of a typical farm field; therefore, the problem has at least two clearly different spatial scales. Application of standard methods to a system like this may bring significant computational difficulties [32]. Besides, in order to calculate the trap count, one has to calculate the population density gradient at the trap boundary. This can be a challenging task, especially at the corner points if the trap has a shape other than circular, and indeed use of traps of various shapes and designs has been increasingly common (cf. [12]). Thus, we have to pay a special attention to numerical issues before discussing ecological results.

The paper is organized as follows. In Section 2.1, we describe the mathematical model and provide a comprehensive analytical study of trap count in the baseline 1D case under various migration scenarios. Although the 1D case is hardly realistic, it provides an important theoretical background for the understanding of a more realistic 2D case. We then briefly revisit the finite-difference method for numerical solution of the diffusion equation and show how it can be used to calculate the trap count in the 1D case (Section 2.2). In Section 3, we carefully test our computational technique against the analytical results in the 1D case. In Section 4, we apply our approach to a hypothetical 1D system in order to reveal generic patterns in the trap count arising from different ecological scenarios. In Section 5, we consider an extension of our method onto the more realistic 2D case and discuss the arising computational issues. We then provide a thorough analysis of trap count for different immigration scenarios by solving the 2D diffusion equation numerically (Section 6). Finally, in Section 7 we discuss the ecological implications of our results.

2. Mathematical model and numerical method, 1D case

2.1. Model

Since the focus of this paper is on the effect of immigration, we neglect the population reproduction, thus assuming that trap counts are collected in the period between the generations. Additionally, for the sake of simplicity, we neglect the population losses due to mortality. The equation describing the population dynamics in space is then essentially the mass conservation law which, in the 1D case, has the following form:

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0, \tag{1}$$

where $u(x,t)$ is the population density at the position x and time t, and j is the population density flux in the direction of axis x. The mathematical description of the flux depends on the type of the individual movement. In a relatively general case, individual insects perform a combination of the non-directed random-like movement that can often be regarded as the Brownian motion [25,41], and a directed movement with a certain speed v. The corresponding population flux is then given by

$$j(x,t) = -D\frac{\partial u(x,t)}{\partial x} + vu(x,t), \tag{2}$$

where D is the diffusion coefficient. Whilst the directed movement becomes important in the presence of environmental gradients, the non-directed random-like motion is an inherent property of almost all ecological populations.

Insect monitoring is done with traps. Once an insect encounters the trap, it is caught with a certain probability $p_0 < 1$ where p_0 depends on the species traits and the trap design. Throughout this paper, we assume that the trap design is sufficiently efficient so that $p_0 \approx 1$. Indeed, this is often the case with *walking insects*, even for a simple pitfall trap design. With regard to the effect of species traits and/or the movement mode, many insects combine flying with walking. Whilst flying is the preferred movement mode when insects travel over long distances (e.g. looking for a new feeding or breeding ground), their movement on the feeding site is typically a combination of walking and very short flights. Correspondingly, here we assume that, once the insects arrive at the farm-field, they mostly move around by walking.

Regarding the trap design, traps can be either baited or non-baited. Baited traps use a certain substance (e.g. pheromone) or agent (e.g. light or color) in order to attract insects to the trap. This introduces an advective component to the insect movement as they are more likely move towards the trap rather than in any other direction. In contrast, non-baited traps do not introduce any directional bias as they capture insects just because of their random encounters with the trap. In this paper, we focus on non-baited traps only; consideration of baited traps involves an essentially different set of assumptions (in particular, about the insect's behavioral response to the attracting agent) and hence will be done elsewhere [3].

Let us consider an idealized 1D farm-field described by the domain $0 < x < L$. We assume that the field is homogeneous and the trap is non-baited. Correspondingly, inside this domain – but not necessarily outside, see below – the monitored insect population performs only random motion, i.e. $v = 0$ in Eq. (2). From Eqs. (1) and (2), we then obtain the diffusion equation:

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2}. \tag{3}$$

The trap, which we assume to be escape-proof, is installed at the left-hand side boundary of the domain, i.e. at $x = 0$. The corresponding condition at the trap boundary is

$$u(0,t) = 0. \tag{4}$$