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Review

Locating multiple tumors by moving shape analysis

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ABSTRACT

A methodology to determine the unknown shape of an embedded tumor is proposed. A functional that represents the mismatch between a measured experimental temperature profile, which may be obtained by infrared thermography at skin surface, and the solution of an appropriate boundary problem is defined. Using the Pennes's bioheat transfer equations, the temperature in a section of healthy tissue with a tumor region is modeled by a boundary problem. The functional is related to the shape of the tumor through the solution of the boundary problem, in such a way that finding the minimum of the functional form also means finding the unknown shape of the embedded tumor. The shape derivative of the functional is computed in each node of an approximation of the solution by the method of Finite Elements using similar methods considered by Pironneau [7]. The algorithm presented include an adaptive strategy to improve the error of the objective function. Numerical results with multiple connected tumors are considered to illustrate the potential of the proposed methodology.

1. Introduction

The modeling of heat transfer in organs has been studied by Pennes since 1948 [2,3]. He suggested that the rate of heat transfer between blood and tissue was proportional to the product of the volumetric perfusion rate and the difference between the arterial blood temperature and the local tissue temperature. Therefore the temperature of a tissue depends on the rate of blood perfusion, the metabolic activity and the heat conduction between the tissue and the environment. The physiological properties of a tumor may differ from a normal tissue, which produces an increase in the temperature of the skin [8–10]. By observing the superficial temperatures of the skin, the location and size of a tumoral tissue may be predicted.

Given the location and shape of a tumor, we can predict the temperature in all the domain. This problem is called the direct problem. The inverse problem consists of using the measurement of the skin temperature to infer the position of the tumor. While the direct problem is well-posed problem [11] (has a unique solution which depends continuously on the data), the inverse problem generally is not.

In [6] the boundary element method was used to locate tumor regions (which were assumed to be elliptical or ellipsoidal) and to find the unknown thermophysical parameters of these regions. In [4] unknown geometrical parameters of an embedded tumor were determined by computing a shape derivative, and solving the differential and adjoint problem with a second order finite difference scheme. In [4] they assumed that the tumor was a spherical simply connected region. The goal of this work is to locate disconnected tumor regions with any shape. We developed an algorithm based on the reaction–diffusion equation, using the finite element method combined with a shape derivative which allows the tumor to change its shape in each iteration. The shape derivative is used to reduce the value of a objective function which measures the difference between the temperature data and its approximation, and is determined using an adjoint method similar to the one introduced in [7]. When a node of the triangulation is moved, it is important to control the error generated by the finite element solution. We developed a posteriori error estimation and use it for refinement and remeshing.

The following equation models the temperature of a skin tissue, $\phi(x)$, that has an embedded tumor region [6]:

$$-\sigma_i \Delta\phi(x) + k_i(\phi(x) - T_b) = q_i(x) \quad i = 1,2 \quad \text{for } x \in \Omega, \tag{1}$$

where Ω represents the two dimensional skin tissue and $\bar{\omega}$ represents the embedded tumor. The sub index $i = 1,2$ identifies the healthy tissue $\Omega - \bar{\omega}$ $(i = 1)$ and the tumor region $\bar{\omega}$ $(i = 2)$ (see Fig. 1), σ represents the thermal conductivity, k is the blood perfusion coefficient, q is the metabolic heat source and T_b is the constant blood temperature. Using the fact that the thermal conductivity, the blood perfusion and the metabolic activity are significantly higher in the tumor region than in normal tissue, we considered that all these coefficients are piecewise continuous. Defining $Q = q + kT_b$, we have the following simplified equation:

$$-\sigma_i \Delta\phi(x) + k_i\phi(x) = Q_i(x) \quad i = 1,2. \tag{2}$$

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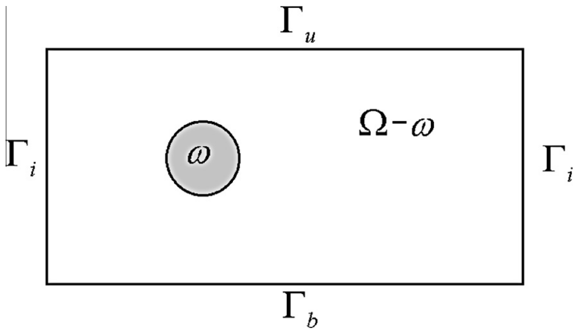

Fig. 1. Two dimensional domain. Ω–ω healthy tissue and ω tumor region.

If we define $\phi_1 = \phi|_{\Omega-\bar{\omega}}$ and $\phi_2 = \phi|_{\omega}$, we have the following boundary conditions:

$$\begin{cases} (i) & \phi_1 = \phi_2 & \text{on } \partial\omega, \\ (ii) & -\sigma_1 \frac{\partial \phi_1}{\partial n} = -\sigma_2 \frac{\partial \phi_2}{\partial n} & \text{on } \partial\omega, \\ (iii) & -\sigma_1 \frac{\partial \phi_1}{\partial n} = \alpha(\phi_1 - T_a) & \text{on } \Gamma_u, \\ (iv) & -\sigma_1 \frac{\partial \phi_1}{\partial n} = 0 & \text{on } \Gamma_i, \\ (v) & \phi_1 = T_b & \text{on } \Gamma_b, \end{cases} \quad (3)$$

where α is the heat transfer coefficient, T_a is the ambient temperature, and n is the outward pointing unit normal. Conditions (3)(*i,ii*) represent the ideal thermal contact between healthy tissue and tumor. A constant core temperature T_b is assumed (3)(v), and a no-flux condition in the lateral boundaries (3)(iv). The convective condition (3)(iii) represents the interchange of temperature between the body and the environment.

The physiological properties of a tumor may produce an increase in the temperature of the skin [8–10], and by observing the superficial temperatures of the skin, the location and size of a tumoral tissue may be predicted. Let ϕ_o be the temperature measured on the boundary Γ_u, which represents the superficial skin, and ϕ_ω the solution of the problem (2) and (3), which depends on the location and shape of ω. The objective function is defined by:

$$E(\omega, \phi_\omega) = \int_{\Gamma_u} |\, \phi_\omega - \phi_o|^2 d\Gamma. \quad (4)$$

The goal is to find $\omega^* \subset \Omega$ such that the respective solution ϕ_* of the problem (2) and (3) verifies: $E(\omega^*, \phi_*) = 0$, which implies that $\phi_* = \phi_o$ in Γ_u. An equivalent way of defining the problem is the following:

$$(P)\begin{cases} \text{Given the following thermophysical constants}: \sigma, k, q, T_b, T_a, \alpha \text{ and the temperature } \phi_o \\ \text{at the boundary } \Gamma_u, \text{ find } \omega^* \subset \Omega, \text{ such that the respective solution } \phi_* \text{ of the problem} \\ (2)-(3) \text{ produces the minimum of the functional } E, \text{ i.e.}: E(\omega^*, \phi_*) = \min_{\omega \subset \Omega} E(\omega, \phi_\omega). \end{cases}$$

2. Variational formulation and discretization of the problem (2) and (3)

The variational formulation of the problem (2) and (3), is the following [4]:

$$\begin{cases} \text{Find } \phi \in H^1(\Omega) \text{ such that} \\ a(\phi, v) = L(v), \quad \forall v \in V(\Omega) \\ \phi = T_b \quad \text{on } \Gamma_b \end{cases} \quad (5)$$

where a, L and $V(\Omega)$ are defined as follows:

$$a(\phi, v) = \int_\Omega (\sigma \nabla\phi \nabla v + k\phi v) dx + \alpha \int_{\Gamma_u} \phi v d\Gamma, \quad (6)$$

$$L(v) = \int_\Omega Q v dx + \alpha T_a \int_{\Gamma_u} v d\Gamma, \quad (7)$$

$$H^1(\Omega) = \{ v \in L^2(\Omega) / \| v\|_{H^1(\Omega)} < \infty \}, \quad \|v\|_{H^1(\Omega)} = \left(\|v\|^2_{L^2(\Omega)} + \|\nabla v\|^2_{L^2(\Omega)} \right)^{1/2}, \quad (8)$$

$$V(\Omega) = \{ v \in H^1(\Omega) : v = 0 \ \text{ on } \Gamma_b \}. \quad (9)$$

We recall that $\|\nabla v\|_{L^2(\Omega)}$ is equivalent to $\|v\|_{H^1(\Omega)}$ in the space $V(\Omega)$.

Let $\{\mathcal{T}_h\}$ be a family of triangulations of Ω such that any two triangles in $\mathcal{T}_h$ have at most a vertex or an edge. For the triangulation $\mathcal{T}_h$, let M be the number of triangles, N the number of nodes, and G the set of index of the nodes which are on the Dirichlet boundary Γ_b. Let h_T stand for the diameter of the triangle $T \in \mathcal{T}_h$, and h the maximum of $h_T, T \in \mathcal{T}_h$. We assume that the family of triangulations $\{\mathcal{T}_h\}$ satisfies the minimum angles condition and, consequently, there exists a constant $\Theta > 0$ such that $h_T/d_T < \Theta$, where d_T is the diameter of the largest circle contained in T. Throughout this work we will denote by c a generic positive constant, not necessarily the same at each occurrence, which may depend on the mesh only through the parameter Θ.

We define the following finite element spaces:

$$V_h^1(\Omega) = \{ w_h \in H_h^1(\Omega) : w_h|_{\Gamma_b} = 0 \} \quad (10)$$

where

$$H_h^1(\Omega) = \{ w_h \in C^o(\Omega) : w_h|_{T_k} \in P^1(T_k) \quad \forall T_k \in \mathcal{T}_h \} \quad (11)$$

$P^1(T)$the space of the polynomials of degree less or equal to 1 in T (12)

We use $H_h^1(\Omega)$ and $V_h^1(\Omega)$ to approximate $H^1(\Omega)$ and $V(\Omega)$, respectively. Then ϕ can be approximated by a function $\phi_h \in H_h^1(\Omega)$ the solution of the problem:

$$\begin{cases} \text{Find } \phi_h \in H_h^1(\Omega) \text{ such that} \\ a(\phi_h, w_h) = L(w_h), \quad \forall w_h \in V_h^1(\Omega) \\ \phi_h = T_b \quad \text{on } \Gamma_b \end{cases} \quad (13)$$

The problem (13) can be solved by finding a solution of a linear matrix system:

$$A\boldsymbol{\phi_h} = F, \quad (14)$$

where

$$A_{ij} = a(\eta_i, \eta_j), \quad F_i = L(\eta_i), \quad (15)$$

$$\boldsymbol{\phi_h} = (\phi_1, \phi_2, \ldots, \phi_N)^T, \quad \phi_h = \sum_{i \notin G} \phi_i \eta_i + \sum_{i \in G} T_b \eta_i, \quad (16)$$

$\{\eta_j\}_{j=1,\ldots,N}$ is a base of $H_h^1(\Omega)$ and $\{q_i\}_{i \in G}$ are the nodes in Γ_b. (17)

It is easy to show that the matrix A is positively defined and symmetric, and therefore a unique solution exists. If $\{q_i\}_{i=1,\ldots,N}$ are the vertex of the triangles of $\mathcal{T}_h$, the base $\{\eta_j\}_{j=1,\ldots,N}$ of $H_h^1(\Omega)$ is uniquely determined by:

$$\eta_i(q_j) = \delta_{ij} \quad \forall i, j = 1, \ldots, N. \quad (18)$$

Then, (4) can be approximated by:

$$E(\omega_h, \phi_h) = \sum_{l \in \Gamma_u} \int_l |\, \phi_h - \phi_o|^2 d\Gamma. \quad (19)$$

We define the following Approximated Problem of (P):

$$(A.P.)\begin{cases} \text{Given the following thermophysical constants}: \sigma, k, q, T_b, T_a, \alpha \text{ and the temperature} \\ \phi_o \text{ in the boundary } \Gamma_u, \text{ find } \omega_h^* \subset \Omega, \text{ such that the respective solution } \phi_h^* \text{ of the} \\ \text{problem (15) gives the minimum of the functional } E, \text{ i.e.}: E(\omega_h^*, \phi_h^*) = \min_{\Omega = \cup T_k} E(\omega_h, \phi_h) \end{cases}$$