Mathematical Biosciences 244 (2013) 201–212

Contents lists available at SciVerse ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

Optimal control of insects through sterile insect release and habitat modification

K. Renee Fister [a], Maeve L. McCarthy [a,*], Seth F. Oppenheimer [b], Craig Collins [c]

[a] Department of Mathematics & Statistics, Murray State University, Murray, KY 42071, USA
[b] Department of Mathematics & Statistics, Mississippi State University, P.O. Drawer MA, MS 39762, USA
[c] Department of Mathematics, The University of Tennessee, Knoxville, TN 37996, USA

ARTICLE INFO

Article history:
Received 17 February 2012
Received in revised form 11 March 2013
Accepted 24 May 2013
Available online 3 June 2013

Keywords:
Ordinary differential equations
Optimal control
Sterile mosquitoes

ABSTRACT

This paper develops an optimal control framework for an ordinary differential equation model to investigate the introduction of sterile mosquitoes to reduce the incidence of mosquito-borne diseases. Existence of a solution given an optimal strategy and the optimal control is determined in association with the negative effects of the disease on the population while minimizing the cost due to this control mechanism. Numerical simulations have shown the importance of effects of the bounds on the release of sterile mosquitoes and the bounds on the likelihood of egg maturation. The optimal strategy is to maximize the use of habitat modification or insecticide. A combination of techniques leads to a more rapid elimination of the wild mosquito population.

1. Introduction

This paper develops a model for the controlled release of sterile insects into an environment where there is an existing population of wild insects. We will also consider the effect of controlling fecundity by altering the environment in such a way that breeding rate is reduced. This activity would take the form of reducing the locations for breeding though removing sources of standing water and of using larvicide or ovacide. We will not consider broad spectrum insecticides because these would also kill our sterilized insects. There has been success in using traps for male insects along with sterile insect release [18], however, we will not consider this third control method in this paper.

The importance of controlling mosquito populations is hard to overstate. It is well known that such diseases as yellow fever, dengue fever, epidemic polyarthritis, Rift Valley fever, Ross River Fever, St. Louis encephalitis, West Nile virus, Japanese encephalitis, LaCross encephalitis, and malaria are carried and transmitted by mosquitoes, [12,26,29,30,34,39,41,42].

This paper considers a model that can applied to many insects, including mosquitoes. Optimal control theory is then applied with a variety of cost functionals to find the best strategy for reducing insect population at minimal cost.

The sterile insect technique was introduced by Knipling [17,18]. The insects are sterilized by irradiation or the application of chemical agents and released to mate with the wild insects. It was used successfully for the screw worm in the late 1950s and early 1960s and great hope was held for using the technique for the control of mosquito populations [19]. Unfortunately, experiments that were carried out with mosquitoes during the same period met with less success. For a discussion of the experimental work in this area see [9,28,38,4].

A number of authors have developed mathematical models of the interaction between sterile and wild mosquitoes, [17,22,3,31]. Some sterile release models have been explicitly connected to particular diseases [7,8,40]. Dumont and Tchuenche [7] consider pulsed sterile release and demonstrate through equilibrium analysis and simulations that frequent small bursts of sterile insects are more effective than larger less frequent releases. Esteva and Yang [8] apply optimal control methods to the rate of introduction of sterile mosquitoes. An approach developed in [40] attempts to control both breeding rates and the rate of introduction of sterile mosquitoes. No bounds have been imposed on the control (s) in any of this work which may be not be realistic biologically.

The use of transgenic insects was developed after the sterile insect technique. Insects carrying a dominant lethal gene are introduced into the population. Alphey et al. [1,2] provide many details of the use of both of these techniques. Models that described the interactions of wild and transgenic mosquitoes include those by Li [23,24] and Diaz et al. [6]. Optimal control methods are applied to the rate of introduction of transgenic mosquitoes by Rafikov et al. [35,36].

* Corresponding author. Tel.: +1 270 8092489.
E-mail addresses: kfister@murraystate.edu (K. Renee Fister), mmccarthy@murraystate.edu (M.L. McCarthy), seth@math.msstate.edu (S.F. Oppenheimer), craig.collins@math.utk.edu (C. Collins).

0025-5564/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.mbs.2013.05.008

It is our hope that by developing new bounded control models for this technique, we may find strategies that will make it more effective.

1.1. The model

We are particularly interested in Li's model of the release of transgenic mosquito populations [24]. Although our focus is on sterile mosquitoes, we will follow the approach in the referenced paper for the model we develop here because it captures the features we seek to incorporate. We consider a population of wild mosquitoes, u, and a population of sterilized mosquitoes, w. If $b(u,w)$ is the birth rate of the wild mosquitoes and $d_u(u,w)$ and $d_w(u,w)$ are the death rates of the wild population and sterilized population respectively, we obtain

$$\frac{du}{dt} = u(b(u,w) - d_u(u,w))$$
$$\frac{dw}{dt} = -wd_w(u,w) + S(t)$$

where S is the release rate of sterile mosquitoes. We will assume the death rate has a constant component and a component that increases with total population density. Thus we will have

$$d_u(u,w) = M + K(u+w)$$
$$d_w(u,w) = M + K(u+w)$$

where the equality of the constants is an implicit assumption of equal fitness between the wild population and the sterilized population. We now turn our attention to the birthrate, $b(u,w)$.

Continuing to follow the approach in [24], we let $c(u,w,t)$ be the number of matings that occur per unit time. Therefore, we can expect that the number of matings of wild type to wild type will be

$$b(u,w) = c(u,w,t)\frac{u}{u+w}$$

This will give us

$$\frac{du}{dt} = u\left(c(u,w,t)\frac{u}{u+w} - M - K(u+w)\right)$$
$$\frac{dw}{dt} = -w(M + K(u+w)) + S(t)$$

Let us consider a couple of choices for the function $c(u,w,t)$. When the total population is large, we expect that mosquitoes will have no difficulty finding a mate, giving us $c(u,w,t)$ as a function only of time, $A(t)$ which is the product of such factors as the likelihood of a mating producing eggs, the (fixed) proportion of the population that is female, the likelihood that an appropriate place can be found so that when the eggs are laid they will hatch, and so on. $A(t)$ can be reduced through the application of larvicide or insecticide, the clearing of breeding sites, etc. Henceforth, we will generally refer to such habitat modification as the application of insecticide, with the understanding that habitat modification can have other features. The function $A(t)$ will serve as a control as well as S, since we are assuming we can take action to reduce the amount of suitable real estate for successful egg laying. This gives the following model

$$\frac{du}{dt} = u\left(\frac{A(t)u}{u+w} - Mu - K(u+w)\right)$$
$$\frac{dw}{dt} = -w(Mw + K(u+w)) + S(t)$$

When the population is relatively small, we expect the law of mass action to be pertinent with $c(u,w,t) = A(t)(u+w)$ where the function $A(t)$ is similar to the function $A(t)$ described above. This gives us

$$\frac{du}{dt} = u(A(t)u - M - K(u+w))$$
$$\frac{dw}{dt} = -w(M + K(u+w)) + S(t)$$

We are particularly interested in a function that can capture the dynamics of both large and small populations simultaneously. We seek a functional form that will lead to approximately the models above. Once again, we follow the work of Li [24] and choose a Holling-II-type functional response, [15]. Fixing a positive constant $\varepsilon > 0$, we set

$$c(u,w) = A\frac{u+w}{\varepsilon+u+w}$$

giving us

$$\frac{du}{dt} = u\left(\frac{Au}{\varepsilon+u+w} - M - K(u+w)\right)$$
$$\frac{dw}{dt} = -w(M + K(u+w)) + S(t)$$

We now rescale, letting $u = \frac{u}{\varepsilon}$ and $w = \frac{w}{\varepsilon}$. Setting $a = A$, $\mu = M\varepsilon$, $\eta = K\varepsilon$, and $s = \frac{S}{\varepsilon}$ yields our final model,

$$\frac{du}{dt} = u\left(\frac{au}{1+u+w} - \mu - \eta(u+w)\right) \tag{1}$$
$$\frac{dw}{dt} = -w(\mu + \eta(u+w)) + s(t). \tag{2}$$

where the initial conditions are

$$u(0) = u_0, \quad w(0) = w_0 \tag{3}$$

and the controls are bounded with $M_1, M_2, N_1, N_2 \geqslant 0$ such that

$$M_1 \leqslant a(t) \leqslant M_2, \quad N_1 \leqslant s(t) \leqslant N_2. \tag{4}$$

The rest of this paper is organized as follows. In Section 2 we establish basic facts about the ODE model. In Section 3, we obtain the existence of an optimal control pair $(a,s)$for different objective functionals. In Section 4 we implement the forward–backward sweep method for each of our cases to obtain numerical results. Finally, in Section 5, we provide discussion of our results and their implications for the optimal control of mosquito populations.

2. Existence

In this section we will obtain the existence, uniqueness, nonnegativity, and boundedness of solutions to our model in a single theorem.

Theorem 2.1. *For nonnegative initial conditions, the model (1), (2) has a unique solution which exists for all time and is nonnegative in each component.*

Proof. Local existence for the system is standard as in [27]. To obtain the result, we first define supersolutions u_1 and w_1 as in

$$\frac{du_1}{dt} = u_1(a - \eta u_1) \quad \frac{dw_1}{dt} = N_2 - \eta w_1.$$

These supersolutions are bounded on a finite interval. Hence, via a comparison result [33], we have that u and w are bounded above on their interval of existence. Moreover, we can let u_2 and w_2 represent subsolutions of the following system,

$$\frac{du_2}{dt} = -Ku_1 \quad \frac{dw_2}{dt} = -Kw_2.$$

where K is a sufficiently large constant. Therefore, we obtain that u and w are bounded below by zero. Consequently, with the coefficients of our original system (1), (2) being bounded, we obtain that