



# Equilibrium population dynamics when mating is by mutual choice based on age



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## ABSTRACT

We consider a steady state model of mutual mate choice in which an individual's mate preferences depend on his/her age, and the preferences are over the ages of prospective mates of the opposite sex. We present a discrete time (and age) model corresponding to successive mating seasons. Males are fertile for  $m$  periods (corresponding to 'age'  $i = 1$  to  $m$ ) and females for  $n \leq m$  periods (they have ages  $j = 1$  to  $n$ ), which is all that distinguishes the sexes. Although we can deal with arbitrary preferences, we concentrate on a simple fertility model where the common utility to a male age  $i$  and female age  $j$  who mate is the number  $K = \min(m - i + 1, n - j + 1)$  of future periods of joint fertility. The incoming sex ratio  $R$  of age 1 males to age 1 females is given exogenously. In each period individuals are randomly (non assortatively) matched and form a mated couple by mutual consent; otherwise they go into the next period unmated and older. We derive properties of equilibrium threshold acceptance strategies and establish the existence of time-invariant age distributions. Our methods determine the age distribution of couples at marriage (mating) and the population sex ratio (OSR) at equilibrium. Since this can be determined empirically in a population, our model can be used to rule out most systems of age preferences (those not consistent with the observed distribution). This extends earlier models of mutual choice with one dimensional types of Alpern and Reyniers [1999. Strategic mating with homotypic preferences. *J. Theor. Biol.* 198, 71–88; 2005. Strategic mating with common preferences. *J. Theor. Biol.* 237, 337–354] where individuals sought, respectively, individuals with similar or high types, but in those models an individual's type was fixed over time.

Under the simple fertility model, at equilibrium the maximum age of an acceptable partner is increasing in the age of the searcher. Our results relate to discussions in the literature regarding optimal parental age differences, age-related mate preferences, and to mate choice in general. We believe our model will be used as a tool in future investigations in these areas.

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## 1. Introduction

It is well known that an individual may have preferences over the age of potential mates of the opposite sex and that these preferences may change as that individual ages. Given such preferences, what sort of mating patterns can we expect at a steady state equilibrium? For a given (say) male, what is the probability that he will end up mating a female of a given age, or of not mating at all. What will be the age distributions of the unmated populations of males and females? We examine such questions in a simplified form where all individuals of a sex are identical except for their age, within the context of mutual mate choice in a steady state

population. (The steady state assumption means that all population parameters remain constant over time, though of course the state of individuals will change.) We assume monogamous mating, where mated individuals never return to the unmated pool after loss (death or otherwise) of their mate. The need for a such a model, where types change over time, was suggested in an earlier paper on mutual mate choice (Alpern and Reyniers, 1999, p.86). Age dependent marital preferences are well known in applied demographic models such as the work of Maitra (2011) on the Indian marriage market.

The papers of Alpern and Reyniers (1999, 2005) presented a dynamic game theoretic analysis of mutual mate choice among heterogeneous individuals distinguished by a single one dimensional type. In the earlier paper, individuals preferred mates with types close to their own; in the latter they preferred high types. This paper takes a player's type to be his/her age, with the consequence

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that one's type is no longer fixed but increases linearly in time from the first fertile period (age 1) to the last (age  $m$  for males and  $n$  for females). We take the maximum age condition  $m \geq n$  as the defining characteristic of the two sexes, so that the males are the sex with the longer period of fertility. As in the earlier papers, we assume non assortative (random) matching of unmated individuals in each period, where matched individuals form a mated couple if they both accept each other. At equilibrium, prospective mates are accepted if they confer a utility greater than the expected utility obtained by going into the next period unmated. A refinement over the previous papers is that we also take as given the numbers  $a_1$  and  $b_1$  of age 1 males and females entering the mating market, and in particular their ratio  $R = a_1/b_1$ , called the *incoming sex ratio*.

Our general model allows for arbitrary preferences, so that the utilities to a male of age  $i$  and to a female of age  $j$  who mate are any  $2mn$  given numbers  $u_{i,j}$  and  $v_{i,j}$ . These numbers fully represent the preferences over time of all the individuals in the model. However in most cases such numbers will be nonsensical, so we restrict our main attention to what we call *fertility models*, where such a couple obtains a *common utility* which is based on their fertilities in their remaining  $K = K(i, j) = \min\{m - i + 1, n - j + 1\}$  periods of joint fertility. This common utility could be interpreted as fitness as measured by the number of offspring. In the *simple fertility model*, we take a binary (fertile/nonfertile) model of fertility and give both partners utility  $K$ . In a jointly optimal mating for a pair of individuals of ages  $i$  and  $j$ , the age difference  $i - j$  will be exactly  $m - n$ , because this is the only pairing in which each member gets a partner with as many remaining years of fertility as he/she has. The 'correct' number of offspring to predict for a given age pair at mating (or age difference in a pair) is the subject of much recent discussion, some of it in a recent volume of Biology Letters (Kokko, 2008a,b; Fielder et al., 2008; Lindqvist et al., 2008). In theory, we could take any of the possibilities mentioned in these papers as our utilities – we have taken  $K(i, j)$  here because it is the simplest. It is possible that further work in our direction could shed some light on that discussion. But as our model allows for arbitrary preferences, we could also model female preferences for older males, possibly argued by positing higher genetic quality for older males due to viability selection. (See Hansen and Price (1995) and Kokko and Lindstrom (1996) for a discussion.) However from a theoretical perspective this argument would be hard to justify in our model, because as there is no mortality, older males are only distinguishable from younger ones in that they have few periods left of fertility. But a quality component of (male) type could be added to our model, together with type-dependent mortality, to consider such questions.

Although our model takes mate preferences as given and deduces from it consequences such as age correlations within mated couples, it can be used to make inferences about mate preferences in an observed population. If the consequences (e.g. age correlations or age at marriage distributions) of a wide variety of possible preference structures are calculated, the observed properties of a population could be matched against these to find which preferences are plausible and which can be ruled out as being inconsistent with our theoretical predictions.

The paper shows how, given the preferences and incoming sex ratio, we can find the equilibrium acceptance strategies, the steady state populations  $a_i$  and  $b_i$  of unmated males and females of age  $i$ , the joint distribution of age at marriage (mating) and the joint distribution of mutually fertile couples. Unlike the pioneering paper of McNamara and Collins (1990), an individual's type is not directly a measure of their attractiveness to the opposite sex, so we do not find the segmentation of types into mutually acceptable bands that they discovered (also found in Alpern and Reyniers, 2005). However our preference relations do have more structure than the independent ones considered by Eriksson et al. (2007b).

As observed by Bergstrom and Real (2000), dynamic modeling of mutual mate choice has received relatively little attention, as compared with one-sided (female choice) models. We trace the beginning of our thread of ideas to Kalick and Hamilton (1986), who examined the 'matching hypothesis' (that people preferred mates similar to themselves) through a simulation study of a cohort, involving mutual choice over a fixed number of periods. A more advanced game theoretic version of a similar cohort model was studied by Johnstone (1997). Related two sided matching problems (involving marriage or otherwise) have been considered by various economists: Burdett and Coles (1997, 1999), Bloch and Ryder (2000), Eeckhout (2000). If one allows transferable utility the modeling is very different, as in Choo and Siow (2006). Though this is a static model, see Choo and Siow (2007) and the survey (Siow, 2008). Mathematicians have studied somewhat more abstract but related models, as in Alpern and Katrantzi (2009), Alpern et al. (2005), Eriksson et al. (2007a), Eriksson and Häggström (2008), Mazalov and Falko (2008) and Ramsey (2008).

Experimental work has sometimes verified our assumption that mate preferences (particularly of females in one-sided choice) vary with age. For example this was found to be true by Kodric-Brown and Nicolletto (2001) for guppies (*Poecilia reticulata*), where different mate preferences were observed for 6-month and 12-month old sexually inexperienced females. More generally, they say "Whether selectivity increases or decreases will depend on an organism's life history, such as age at first maturity, age-related changes in fecundity and expected life span, all of which should influence mate choice". We show for our fertility model that selectivity decreases over time, agreeing with human studies carried out by Pennebaker et al. (1979). They observe the problem of distinguishing between preference changes due to age itself and those due to mating experience (we consider only the former, as there is no benefit of learning in our model). It has also been observed, for example by Gibson and Høglund (1992), that younger females may imitate the choices of older females, something that might be relevant if we amended our model so that (say) a male's type was not fully observable.

The paper is organized as follows. Section 2 describes our discrete time model for arbitrary age preferences and then specializes to the simple fertility model. In Section 3, general properties of equilibrium threshold strategies are derived and the existence of invariant age distributions is established. Section 4 gives examples of the equilibria we obtain for small values of  $m$  and  $n$ , for arbitrary incoming sex ratios  $R$ . We show that for some intervals of  $R$  there may be no pure strategy equilibria; only a mixed strategy equilibria, in which a male  $i$  may accept some female age  $j$  with a nontrivial probability. Section 5 is a short Conclusion.

### 1.1. Contrast with cooperative game theory approach of 'stable matchings'

The model analyzed in this paper is a decentralized mating model using non cooperative game theory, particularly a solution concept analogous to subgame perfect Nash equilibrium. It is a dynamic game in which each period first randomly generates a *matching* (bijection between unmated females and a similar sized random set of unmated males) and then a *mating* (restriction of the above bijection) is formed between matched pairs who mutually accept.

This is in stark contrast with the centralized mating process (usually called the stable matching problem, but the word matching would be confusing here) which involves a central processor who knows the preferences of each individual over prospective mates of the opposite sex. A bijection between the equal sized female and male populations is sought in which no male–female pair prefer each other to their current partners. Such processes are very

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