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Application of critical path analysis for permeability prediction in natural porous media

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ABSTRACT

Critical path analysis (CPA), originally developed to describe electrical conductance in semiconductors, has been shown recently to hold some promise in describing transport properties of porous media. I applied some previously developed concepts in CPA and percolation theory to predict permeability in a suite of sandstone, carbonate, and clay-rich samples. I assumed that the pore sizes in my samples exhibited fractal scaling and expressed the electrical formation factor as a function of porosity using universal scaling from percolation theory. The resulting CPA permeability predictions match the measured values very well. In addition, I show how considering the scale-dependence of the percolation threshold yields two characteristic length scales for transport properties: the critical pore size, and the sample size. This work suggests that the CPA framework is appropriate for describing transport properties of natural porous media, and provides a theoretical basis for understanding the permeability of tight rocks like shale in which laboratory permeability measurements are difficult.

1. Introduction

Permeability, an intrinsic property of a porous medium that describes the ease with which fluid may be transmitted through the medium, is an important parameter to quantify for any application involving fluid flow or pore pressure quantification. The relationship between porous medium microstructure and permeability has been an area of active research for some time. This relationship is an important component of predicting permeability from other physical properties (Bear, 1972; Dullien, 1992). Such methods of permeability prediction are particularly important in low-permeability media like shales and tight gas sandstones, in which laboratory measurements of permeability are extremely difficult and time-consuming (e.g., Cui et al., 2009). In idealized porous media, permeability may be determined from pore or grain geometry according to analytical solutions to the Navier-Stokes equations (e.g., Dullien, 1992; Happel and Brenner, 1983). However, natural porous media typically exhibit a wide range of pore and grain shapes, which makes permeability determination much more difficult. Recent work has also demonstrated that permeability may be determined by direct numerical simulation of flow in three-dimensional digital images of porous media (e.g., Bultreys et al., 2016; Meakin and Tartakovsky, 2009). These methods may suffer from resolution limits in media with pores smaller than a few microns, and computational issues in simulating flow in a volume large enough be representative, but do represent a good alternative for cross-validation with other methods in most media. To overcome the limitations of calculations in idealized media without invoking the technological complexity of direct numerical simulation, researchers have turned to percolation theory, which can allow determination of bulk transport properties from structural characteristics of the pore system (e.g., Berkowitz and Balberg, 1993). One particular technique based on percolation theory, critical path analysis (CPA), has shown promise in determining permeability in strongly heterogeneous media like rocks and soils in which fluid transport occurs only through a small subset of the highest-conductance pores (Friedman and Seaton, 1998; Hunt, 2001; Skaggs, 2011).

According to CPA, a heterogeneous porous medium is assumed to be composed of flow pathways (pores) with different conductances – that is, different abilities to transmit flow. The conductance is a function of the pore size, with larger pores having larger conductances. Most flow through the medium occurs on high-conductance pathways that comprise a fraction of the total volume available for flow. The macroscopic flow is therefore dictated by the low-conductivity restrictions that occur along these pathways (Hunt, 2001). CPA was originally used to describe electrical conductivity in amorphous germanium, silicon, and carbon used in semiconductors (Ambegaokar et al., 1971; Pollak, 1972). Subse-

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quent researchers have applied CPA to both electrical conductivity and permeability in natural porous media (Bernabé and Bruderer, 1998; Charlaix et al., 1986; Charlaix et al., 1987; Friedman and Seaton, 1998; Hunt, 2001; Hunt and Gee, 2002; Skaggs, 2011). According to this application of CPA, conduction (either electrical or hydraulic) occurs mainly through the higher-conductivity (i.e., larger) pores in the medium. Let $f(r)$ be the volumetric probability density function of pore sizes in the medium. In cylindrical pores, the pore size r represents the pore radius. A critical pore size r_c may be defined as

$$\int_{r_c}^{\infty} f(r)dr = p_c, \tag{1}$$

where p_c is the percolation threshold (fraction of pore volume that must be occupied for a sample-spanning, connected cluster of pores to form). The critical pore size represents the smallest restriction along the sample-spanning cluster (e.g., Robins et al., 2016). The conductance in a pore of size r_c is the critical conductance g_c (Ambegaokar et al., 1971; Hunt, 2001). If p_c and $f(r)$ are known, the permeability may be estimated from the critical hydraulic conductance, but the relationship depends on the size distribution and connectivity of the pore system (Halperin, 1989).

The problem of predicting permeability from the distribution of hydraulic conductances may be alleviated by considering electrical conductance together with hydraulic conductance, as the relationship between the macroscopic conductances and the pore-scale distribution of conductances is assumed to be the same in both cases (Hunt, 2001; Skaggs, 2011). Electrical and hydraulic conductances generally exhibit different relationships with pore size, but by assuming that hydraulic conductance is described by Poiseuille's law and electrical conductance by Ohm's law, the ratio of the critical electrical and hydraulic conductances (g_c^e and g_c^h) in a cylindrical pore is proportional to ${r_c}^2$ (Banavar and Johnson, 1987; Friedman and Seaton, 1998; Hunt, 2001; Skaggs, 2011), which provides a relationship between permeability and electrical conductivity. Based on this proportionality, several CPA-based permeability models have been published in the literature that include a relationship with electrical conductivity (Bernabé and Bruderer, 1998; Friedman and Seaton, 1998; Hunt, 2001; Le Doussal, 1989; Skaggs, 2011), and this treatment is consistent with other permeability relationships that use electrical conductivity but are not explicitly derived using CPA (Avellaneda and Torquato, 1991; Bernabé and Revil, 1995; Daigle and Reece, 2015; Johnson et al., 1986; Katz and Thompson, 1986; Revil and Cathles, 1999; Revil and Florsch, 2010).

CPA has been shown to be successful in predicting the saturation-dependence of permeability in soils (Ghanbarian-Alavijeh and Hunt, 2012a; Hunt, 2001; Hunt and Gee, 2002), but its applicability to predicting permeability under fully saturated conditions has not been fully tested in laboratory samples. Skaggs (2011) summarized the existing doubts about the applicability of CPA to porous media as (1) is the localized transport through the largest pores described by Eq. 1 applicable to porous media? And (2) is the significance of the critical pore size due to the localized transport predicted by CPA, or merely due to a coincidental relationship with other length scales? I analyzed experimental data on Berea sandstone, Racine dolomite, and a suite 13 of hemipelagic marine mud samples from three boreholes in the Nankai Trough offshore Japan, and compared permeability predictions from CPA with laboratory measurements. I found that CPA was able to predict permeability remarkably well over nearly 4 orders of magnitude by combining the method of Friedman and Seaton (1998), which determines permeability from the ratio g_c^h/g_c^e, with a percolation theory-based description of electrical conductivity that allows extension of the Friedman and Seaton method to media with significant surface conductivity. My results suggest that (1) the localized transport prediction of CPA is applicable to natural porous media as long as the pore size distribution spans more than roughly 2.6 orders of magnitude; (2) while r_c is proportional to other important length scales for transport in porous media, r_c and the size of the medium are the two fundamental length scales that describe transport through the pore space; and (3) considering both r_c and the size of the medium results in an improved permeability prediction compared to consideration of r_c alone as done in previous models (e.g., Katz and Thompson, 1986).

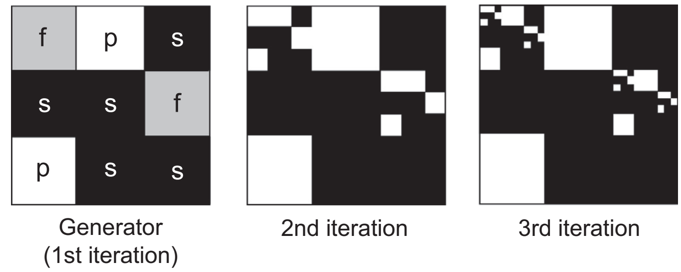

Fig. 1. Simple illustration of pore-solid fractal model in two dimensions with square pores. The fractal generator is shown at left. Solid material is marked "s", pores are marked "p", and areas where the generator is to be repeated are marked "f". This generator has $\beta = p/(p+s) = 2/7$. The middle panel shows the second iteration, in which the generator has been repeated in each of the squares marked "f". The panel at right shows the third iteration.

2. Theory

2.1. Fractal parameterization of the pore size distribution

In investigations of CPA, authors have used several different methods of parameterizing the pore size volumetric probability density function, including lognormal, log-uniform, and power-law distributions. In the present work I use a fractal description of the pore space, which results in a power-law distribution. The description of the pore space using a fractal model implies that the pore system displays statistical self-similarity (Tyler and Wheatcraft, 1990), which according to Mandelbrot (1967) means "each portion can be considered a reduced-scale image of the whole." My choice is informed by the arguments of Hunt, (2001), Hunt and Gee, (2002), and Sahimi, (1993), who all presented evidence that pore sizes in natural porous media tend to follow fractal scaling with finite upper and lower bounds and that the fractal parameterization allows for successful prediction of saturated and unsaturated permeability. I use a particular model, the pore-solid fractal (PSF) model (Bird et al., 2000; Perrier et al., 1999), whose parameters all have a physical interpretation and which has been used to predict saturation-dependent permeability in previous work (Ghanbarian-Alavijeh and Hunt, 2012a).

The PSF model considers the porous medium to be composed of three components: pores, solids, and volume in which the fractal generator is replicated iteratively (Fig. 1). The following derivation is given in Hunt and Gee, (2002) and Ghanbarian-Alavijeh and Hunt, (2012a), and previous work (e.g., Bird et al., 2000; Fisher, 1971; Ghanbarian-Alavijeh and Hunt, 2012a) has shown its applicability to cylindrical pores. If the cumulative distribution of pore sizes follows $N(<r) \sim r^{-D}$ where $N(<r)$ is the number fraction of pores smaller than size r and D is the fractal dimension, in three dimensions, the PSF model expresses the volumetric probability density function $f(r)$ as

$$f(r) = \frac{\beta}{\phi}\frac{3-D}{r_{\max}^{3-D}} r^{2-D}, \tag{2}$$

where Φ is porosity, $r_{\max}$ is the largest pore size in the medium, and β is the ratio of pore volume to the sum of the pore and solid volumes in the fractal model (Fig. 1); note that $\beta \geq \phi$ because β