Advances in Water Resources 96 (2016) 423–437

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

A Robust volume conservative divergence-free ISPH framework for free-surface flow problems

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ARTICLE INFO

Article history:
Received 28 January 2016
Revised 17 August 2016
Accepted 22 August 2016
Available online 23 August 2016

Keywords:
Flow through porous media
Incompressible smoothed particle hydrodynamics
Density current
Volume conservation

ABSTRACT

This study presents a Volume Conservative approach for resolving volume conservation issue in divergence-free incompressible Smoothed Particle Hydrodynamics (ISPH). Irregular free surface deformation may introduce error in volume computation, which has a cascading effect over time. Proposed correction decreases this numerical compressibility to a minimal value. The correction is obtained directly by solving Navier-Stokes momentum equation. Consequently, the framework does not require any parametric study for mixed source/sink term or iterative solution of pressure Poisson equations. The correction is implemented on four different types of flow: (a) pressurized flow in a closed box, (b) dambreak flow, (c) flow through porous block, (d) lock-exchange flow of immiscible fluids (both free-surface and pressurized flow). All four scenarios are shown to have minimal error compared to pure divergence-free ISPH.

1. Introduction

Free surface flow problems are common in natural systems. Underlying physics of the problem becomes difficult to model in presence of free surface boundary. Shallow-water equations use depth-averaged approach to compute hydrodynamic variables. The fluctuation in pressure and vertical velocity cannot be captured precisely by Eulerian frameworks (Pahar and Dhar, 2014) with shallow-water equations. In recent years, mesh-free Smoothed Particle Hydrodynamics (SPH) has become popular for free-surface flow problems. However, SPH suffers some drawbacks in handling problems with incompressibility. Pure incompressibility or divergence-free continuity equation does not consider any density change. Numerical compressibility may appear for pure incompressible SPH model. The error in representative volume increases steadily over time. This may result in unrealistic value. A Volume Conservative ISPH (VC-ISPH) algorithm is proposed to overcome the compressibility problem.

SPH was derived by Gingold and Monaghan (1977), and Lucy (1977) for modelling of astrophysical problems. Later, Monaghan (1994) introduced SPH in free surface flow simulation. Pressure field obtained by standard weakly compressible SPH may contain spurious variation. Various correction and renormalization algorithms need to be applied to get smoothed pressure and density field (Gomez-Gesteira et al., 2010). Incompressible SPH can produce comparatively better result with larger time step (Lee et al., 2008). Incompressibility in SPH is mainly achieved by using a semi-implicit pressure projection scheme (Hosseini et al., 2007). Pressure is derived implicitly by solving Pressure Poisson Equation (PPE). The source/sink term of PPE differs with choice of scheme. Cummins and Rudman (1999) proposed that the source/sink term should contain divergence of intermediate velocity. Their algorithm of solenoidal velocity field provides smooth pressure profile. However, particles tend to proceed along streamlines in pure divergence-free Lagrangian method (Shadloo et al., 2012). An artificial displacement term needs to be added at the end of each time step for removing tensile instability (Xu et al., 2009). Shao and Lo (2003) replaced the source/sink term with intermediate density based on particle position. Density invariance schemes are very effective to conserve proper volume of the fluid considering the sensitivity of density in SPH. The scheme also does not require any artificial displacement to maintain stability. However, purely divergence-free velocity field may not be obtained (Gui et al., 2015). On the other hand, divergence free schemes consider density to be constant at all time levels. Kernel compactness algorithm is not directly utilized unlike density-invariant schemes. Consequently, small errors in density computation may be induced. As no compactness checking algorithm is utilized, the errors may accumulate over time to obtain an infeasible value. Thus, total volume of fluid may not be conserved by the scheme. Various researchers tried to create hybrid schemes to balance both diver-

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http://dx.doi.org/10.1016/j.advwatres.2016.08.010
0309-1708/

gence free and density invariance criteria. Hybrid schemes mainly solves PPE with combined source term containing weighted average of intermediate velocity and density. These schemes are mainly different variants of density invariance scheme. Asai et al. (2012) derived the weight factors based on hydrostatic pressure test. Principle of energy dissipation and height-depth ratio of the flow system were used by Gui et al. (2014) for calculating weight factor. A constant value of 0.5 was suggested by Koh et al. (2013) for sloshing problem. In Moving particle Semi-implicit method, an improved higher order source/sink term was proposed by Khayyer and Gotoh (2011). Hu and Adams (2007) solved multiple poisson equations in a single time step until a suitable convergence criteria is achieved. Their method was later extended (Hu and Adams, 2009) by solving two consecutive PPEs containing divergence-free and density invariant source/sink terms in half-time steps. Very few studies (Gui et al., 2015) have concentrated on quantification of errors in terms of density and velocity-divergence for free-surface flow simulation. However, purely divergence-free volume-conservative approach including multiphase flow through porous media is not reported in the literature.

This study presents a volume conservative divergence free incompressible SPH. A corrective displacement is added at the end of each time step. The correction is derived based on total incompressibilty of continuity equation. Divergence-free incompressible SPH algorithm is adopted as base algorithm due to its capability of conserving solenoidal criteria. The corrective framework can be applied with divergence-free ISPH without any parametric study. It also does not need iterative solutions of multiple PPE within a single time-step. Current model is validated for a lid-driven cavity flow with Reynolds number 1000. The proposed model is applied on three two-dimensional (2D) representative numerical examples. Change of volume is measured against basic ISPH algorithm (Xu et al., 2009) for each case.

2. ISPH Model for multiphase flow and flow through porous media

2.1. Governing equations

For divergence-free incompressible SPH, governing equation needs to be represented in terms of material derivative. Different forms of integrated flow models can be modelled in ISPH, provided the momentum equation contains material derivative of velocity. Cheng et al. (2012) derived an integrated equation for free-surface flow interaction with porous media for Eulerian framework. Generalized Lagrangian form of governing equations for multiphase flow including flow through porous media can be represented as,

$$\nabla.\mathbf{u} = 0 \tag{1a}$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{\eta}{\rho}\nabla P + \eta\mathbf{g} + \nu\nabla^2\mathbf{u} + \mathbf{F}_R \tag{1b}$$

$\mathbf{u}=$ Darcy velocity vector, $P=$ pressure, $\rho=$ density of the fluid, $\mathbf{g}=$ acceleration due to gravity, $\nu=$ kinematic viscosity, $\eta=$ effective porosity inside porous media, $\mathbf{F}_R=$ resistive force ($\mathbf{R}/\rho$) due to presence of porous media and divergence of residual stress tensor.

For pure free-surface multiphase flow through non-porous region, η becomes unity. $\mathbf{F}_R$ will represent $\frac{1}{\rho}\nabla.\bar{\tau}$, where $\bar{\tau}$ is Sub-Particle-Scale (SPS) tensor. Detailed derivation of the governing equations for both pure fluid and porous regions are presented in Appendix.

2.2. Smoothed particle hydrodynamics(SPH)

In SPH, fluid domain is considered to be constituted of a scattered set of nodes/particles. Each particle has a circular support domain with radius $2h$. Every particle will interact with the neighboring particles present in the support domain. The amount of interaction is determined by kernel/weight (w) function. Value of kernel decreases with distance from the centre particle and outside support domain it is zero.

$$f(\mathbf{r}) = \int f(\mathbf{r}')\delta(|\mathbf{r}-\mathbf{r}'|)d\mathbf{r}' \tag{2}$$

Dirac delta function is substituted with a distributed kernel.

$$f(\mathbf{r}) = \int f(\mathbf{r}')w(|\mathbf{r}-\mathbf{r}'|, h)d\mathbf{r}' \tag{3}$$

Integral formulation can be expressed as summation of discrete quantities in support domain.

$$f(\mathbf{r}) = \sum_b f_b w(|\mathbf{r}-\mathbf{r}_b|, h)\nu_b \tag{4}$$

such that

$$\sum_b w(|\mathbf{r}-\mathbf{r}_b|, h)\nu_b = 1 \tag{5}$$

where $\nu_b=$ volume associated with particle b.

2.3. Operators

Instead of using basic kernel gradient operators, velocity and pressure operators are computed using simple algebraic calculations. Velocity divergence of a particle can be written as,

$$\nabla\mathbf{u}|_a = \sum_b \frac{m_b}{\rho_b}(\mathbf{u}_b - \mathbf{u}_a).\nabla w_{ab} \tag{6}$$

To conserve linear momentum, pressure gradient can be estimated by,

$$\frac{\nabla P}{\rho}|_a = \sum_b m_b\left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2}\right)\nabla w_{ab} \tag{7}$$

where $m_b=$ mass of particle b and $\rho_b=$ density of particle b. Adami et al. (2012) has given a modified pressure gradient for simulating multiple fluids with different density.

$$\frac{1}{\rho}\nabla P|_a = \frac{1}{m_a}\sum_b (\nu_a^2+\nu_b^2)\tilde{P}_{ab}\nabla w_{ab} \tag{8}$$

where $\tilde{P}_{ab}=$ density weighted interparticle averaged pressure. Hu and Adams (2006) proposed that the interparticle average pressure can be expressed as,

$$\tilde{P}_{ab} = \frac{\rho_a P_b + \rho_b P_a}{\rho_a + \rho_b} \tag{9}$$

Near free-surface, particles do not have full kernel support. The kernel derivative can be corrected by using algorithm proposed by Bonet and Lok (1999). Kernel second order derivatives are computed using a combination of first-order kernel derivatives (Morris et al., 1997).

$$\frac{1}{\rho}\nabla.(\nabla P)|_a = \sum_b 2\frac{\nu_b}{\rho_b}\frac{(P_a-P_b)\mathbf{r}_{ab}.\nabla w_{ab}}{|\mathbf{r}_{ab}|^2+\chi} \tag{10}$$

To avoid zero in denominator, a very small coefficient χ is added($0.0001h^2$). Quintic Wendland (1995) is adopted for computing non-linear flow phenomena.

$$W(r,h) = \frac{7}{4\pi h^2}\left(1+\frac{r}{h}\right)\left(1-\frac{r}{2h}\right)^4, \quad \forall r \in [0, 2h] \tag{11}$$

2.4. Resistive force

Resistive force term ($\mathbf{F}_R$) denotes an extra linear and non-linear drag forces inside porous media. In pure fluid domain, the term represents only $\frac{1}{\rho}\nabla.\bar{\tau}$, where $\bar{\tau}$ is Sub-Particle-Scale (SPS) tensor.