Advances in Water Resources 97 (2016) 38–51

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

A practical model for fluid flow in discrete-fracture porous media by using the numerical manifold method

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ARTICLE INFO

Article history:
Received 3 April 2016
Revised 1 September 2016
Accepted 1 September 2016
Available online 3 September 2016

Keywords:
Fluid flow
Discrete fractures
Porous media
Non-conforming mesh
Numerical manifold method

ABSTRACT

In this study, a numerical manifold method (NMM) model is developed to analyze flow in porous media with discrete fractures in a non-conforming mesh. This new model is based on a two-cover-mesh system with a uniform triangular mathematical mesh and boundary/fracture-divided physical covers, where local independent cover functions are defined. The overlapping parts of the physical covers are elements where the global approximation is defined by the weighted average of the physical cover functions. The mesh is generated by a tree-cutting algorithm. A new model that does not introduce additional degrees of freedom (DOF) for fractures was developed for fluid flow in fractures. The fracture surfaces that belong to different physical covers are used to represent fracture flow in the direction of the fractures. In the direction normal to the fractures, the fracture surfaces are regarded as Dirichlet boundaries to exchange fluxes with the rock matrix. Furthermore, fractures that intersect with Dirichlet or Neumann boundaries are considered. Simulation examples are designed to verify the efficiency of the tree-cutting algorithm, the calculation's independency from the mesh orientation, and accuracy when modeling porous media that contain fractures with multiple intersections and different orientations. The simulation results show good agreement with available analytical solutions. Finally, the model is applied to cases that involve nine intersecting fractures and a complex network of 100 fractures, both of which achieve reasonable results. The new model is very practical for modeling flow in fractured porous media, even for a geometrically complex fracture network with large hydraulic conductivity contrasts between fractures and the matrix.

1. Introduction

Fractures are ubiquitous in natural geological porous media as products of folding, faulting, subsidence, thermal deformation, or other geological activities over time. Fractures may act as conduits or seals for fluid flow, and their size ranges from microns to kilometers. Analyzing flow in naturally fractured porous media is crucial in a wide range of geological and environmental engineering applications, including hydrocarbon and geothermal reservoir engineering, nuclear waste disposal, CO2 sequestration, and ground water management (Berkowitz, 2002; Bonnet et al., 2001; MacMinn et al., 2010).

The methods to simulate flow in fractures are categorized into the following types depending on the heterogeneity of the fractured porous media: (1) equivalent continuum models, (2) discrete fracture models, (3) their hybrid (Berkowitz, 2002; Barker, 1991; Neuman, 2005; Pinder et al. 1993; Sahimi, 1995) or (4) dual-continuum models (Barenblatt et al., 1960; Pruess, March 1983). In the aforementioned applications, the presence of arbitrarily oriented and intersecting fractures may influence the flow in a complex manner that cannot be simplified with homogenization. Therefore, modeling flow in discrete fractures is very important. Among existing discrete fracture models, two approaches have been developed that depend on the permeability of rock matrix. If the permeability of the rock matrix is negligible, fluid flow is assumed to only occur within a connected fracture network (Adler and Thovert, 1999; Committee on Fracture Characterization and Fluid Flow 1996; Zhang and Yin, 2014; Berrone et al., 2014; Hyman et al., 2015). The other approach considers fluid flow in both the rock matrix and fractures, i.e. the discrete-fracture-matrix model, in which the rock matrix may act the primary medium for flow when the fractures are sealed. In this study, we focus on the modeling of fluid flow in discrete fractured porous media in 2D by considering a permeable rock matrix as porous media.

The greatest challenges of modeling fluid flow in discretely fractured porous media are the complex geometry of fractures, the large contrasts in geometries and the hydraulic properties between

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http://dx.doi.org/10.1016/j.advwatres.2016.09.001
0309-1708/

fractures and the rock matrix. One approach to model such a problem is to use the same dimension for fractures as for the rock matrix in an equi-dimensional formulation (Belayneh et al., 2006; Caillabet et al., 2000; Kim and Deo, 2000; Pruess et al., 1990; Reichenberger et al., 2006; Teimoori et al., 2005). The authors developed a similar approach for dominant fractures as a heterogeneous medium (Hu et al., 2015). When fractures densely intersect, high computational load and convergence problems may appear because of small integration areas. Another formulation that is based on the concept of reduced dimension has been proposed to avoid such issues (Hoteit and Firoozabadi, 2005; Juanes et al. 2002; Karimi-Fard et al., 2004; Alboin et al., 2000; Angot et al., 2009; Bogdanov et al., 2003a; Bogdanov et al., 2003b; Bogdanov et al., 2007; Bogdanov et al., 2003c; Mourzenko et al., 2011; Adler et al., 2012; Martin et al., 2005; Carlo and Scotti, 2012). The basic idea of a reduced-dimension formulation is to represent the fractures with an $n-1$ dimensional mesh in an n-dimensional problem. A variety of numerical methods have been developed based on the reduced-dimension idea, including finite difference, finite volume and finite element methods (Hoteit and Firoozabadi, 2005; Juanes et al., 2002; Karimi-Fard et al., 2004; Alboin et al., 2000; Angot et al., 2009; Bogdanov et al., 2003a; Bogdanov et al., 2003b; Bogdanov et al., 2007; Bogdanov et al., 2003c; Mourzenko et al., 2011; Adler et al., 2012). In these models, the fractures are usually represented in a mesh by interfaces between matrix elements, which requires that the mesh conform to the fractures. For example, a series of 3D finite-volume models was applied to analyze well tests (Bogdanov et al., 2003a), single-phase (Bogdanov et al., 2003b; Bogdanov et al., 2007), two-phase (Bogdanov et al., 2003c), and transient compressible flow (Mourzenko et al., 2011; Adler et al., 2012) under the condition that a robust meshing algorithm is provided. Non-conforming-mesh methods for fluid analysis in fracture-matrix systems, in which the mesh does not have to conform to the fracture geometries, were developed based on mixed (Martin et al., 2005) and extended (Carlo and Scotti, 2012) finite element methods. However, in such an extended finite element formulation, an enriched approximation of fractures may limit the applicability of modeling complexly intersecting fractures, similar to problems that are related to mechanical analysis, as discussed by An et al., (2012). Therefore, an improved non-conforming-mesh method that can achieve high accuracy and deal with complexly intersecting fractures is required.

The numerical manifold method (NMM) is a promising numerical method for modeling mechanical processes in continuous/discontinuous media (Shi, 1991; Shi, 1996). The establishment of the NMM was motivated by the urgent need to analyze dynamic processes that span both continuous and discontinuous media in a unified approach, which is not possible with continuous methods such as the finite element method (FEM) or discontinuous methods such as the distinct element method. The NMM has indeed been successfully applied to rock mechanics analyses of both continuous and discontinuous geologic media (Ma et al., 2010; Ma et al., 2009; Zheng and Xu, 2014). The numerical grids (or meshes) in the NMM consist of mathematical and physical covers. Mathematical covers overlay the entire material domain and determine the approximation's precision by the mesh density. The physical covers are divided from the mathematical covers by boundaries and discontinuities (such as fractures) and determine the integration domain. The overlapping parts of physical covers form elements. The generation of physical covers and elements can be conveniently realized by a one-time tree-cutting algorithm (Shi, 1988). Each of the divided physical covers has its independent local function. The global approximation of an element is the weighted average of the cover function that overlaps the element. Based on this definition framework, the NMM appears to be a promising method for modeling flow in discontinuous media and superior to continuous methods. Specifically, the following features of the NMM can be highlighted as particularly advantageous for solving discontinuous/continuous fluid flow problems: (1) the meshing effort can be substantially reduced with non-conforming mathematical covers; (2) the physical covers that are divided by a fracture have independent functions, which represent the discontinuous nature of the fractures and do not require enrichment functions; (3) the intersection of fractures can be realized by a tree-cutting algorithm and explicitly represented by divided physical covers and elements; and (4) the simplex integration that is used in the NMM (Shi, 1996) achieves exact solutions in polygons with complex shapes. For fluid flow modeling using NMM, Ohnishi et al., January (1999) developed a node-shifting method for saturated-unsaturated flow modeling with the NMM. In Hu et al., (2015), the authors compared two methods for modeling heterogeneous flow, including flow in equi-dimensional fractures based on an energy-work seepage model (Wang et al., 2014). Recently, a mixed head- flux formulation was also proposed and applied to unconfined fluid flow in porous media (Zheng et al., 2015).

In this study, we develop a new model to analyze flow in porous media that contain discrete fractures by using the NMM with a non-conforming mesh. First, we introduce the fundamentals of the NMM in Section 2, including the fundamental definitions, the global approximation, and the tree-cutting algorithm to generate the mesh from a given discrete fracture network geometry. In Section 3, we present the development of the new model for flow along and normal to fractures without introducing additional degrees of freedom for the fractures. In Section 4, we verify the new model with some examples and compare the results to the analytical solutions. In Section 5, the new model is applied to two problems with densely intersecting fractures. In Section 6, we discuss some potential extensions of the current model to 3D fluid flow analysis and to coupled hydro-mechanical analysis for discrete-fracture porous media.

2. Fundamentals of the NMM

In this section, we introduce the fundamentals of the NMM, including the NMM's approximation for fluid flow, the relationship of the NMM to other methods such as the finite element method (FEM) and distinct deformation analysis (DDA), and the generation of discrete fracture with a tree-cutting algorithm.

2.1. NMM approximation

The NMM, which was proposed by Shi, (1991), is based on the concept of "manifold" in mathematical topology. In the NMM, independent mathematical and physical covers are defined. The mathematical covers are user-defined and consist of finite overlapping covers that occupy the entire material domain. Regular meshes such as finite difference grids, finite elements or convergence regions of series can be used as mathematical covers. These mathematical covers determine the numerical precision by the mesh shape and mesh density. For example, equilateral triangular mathematical covers perform better than rectangular covers in terms of the precision and stability of the solution, as derived and proven by examples in 2D mechanical analysis (An et al., 2011). On the other hand, the numerical precision increases with a denser approximation mesh (i.e. the mathematical mesh in the NMM) to a certain extent. The physical covers are mathematical covers that are divided by boundaries, fractures or discontinuities, which determine the integration fields. A local approximation is assigned on each physical cover, for example, a constant (simplified, as in the FEM), linear function or series. The weighted average of the local cover functions forms the global approximation. Therefore, the NMM is flexible and general enough to include and