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A lattice Boltzmann study on the impact of the geometrical properties of porous media on the steady state relative permeabilities on two-phase immiscible flows

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ABSTRACT

In the current paper, the effect of the geometrical characteristics of 2-D porous media on the relative permeability in immiscible two-phase flows is studied. The generation of the different artificial porous media is performed using a Boolean model based on a random distribution of overlapping circles/ellipses, the size and shape of which are chosen to satisfy the specific Minkowski functionals (i.e. volume fraction, solid line contour length, connectivity). The study aims to identify how each different Minkowski functional affects the relative permeability of each phase at various saturations of the non-wetting phase. A 2-D multi-relaxation time (MRT) lattice Boltzmann model (LBM) that can handle high density ratios is employed in the simulation. The relationship between the driving forces G and the relative permeabilities of the two phases for every artificial structure is quantified. It is found that for high non-wetting phase saturations (fully connected flow), a non-linear relationship exists between the non-wetting phase flow rate and the driving force, whilst this relationship becomes linear at higher magnitudes of the latter. The force magnitude required to approach the linear region is highly influenced by the pore size distribution and the connectivity of the solid phase. For lower non-wetting phase saturation values, its relative permeability in the linear regime decreases as the fraction of small pores in the structure increases and the non-wetting phase flow becomes disconnected. A strong influence of the solid phase connectivity is also observed.

1. Introduction

The understanding of immiscible two-phase flows in porous media is of critical importance in industrial operations such as, enhanced oil recovery, geologic CO_2 sequestration, groundwater supply and remediation, catalytic processing in fixed bed reactors etc. Conventional macro-scale multiphase flow models in porous media rely heavily on phenomenological extensions of Darcy's law, where each phase moves through its own channel which is bounded only by the solid walls. The viscous coupling between the two phases is ignored in this kind of approach. Hence, the superficial velocity of each phase is described as:

$$\boldsymbol{v}_i = -\frac{\kappa \kappa_{r,i}}{\mu_i} \nabla p_i, \quad i = w, nw \tag{1}$$

where $\boldsymbol{v}_i$, ∇p_i and μ_i are the superficial velocity, pressure gradient and dynamic viscosity of fluid i respectively. The subscripts w and nw denote the wetting and the non-wetting phase respectively, κ is the intrinsic permeability of the porous medium and $\kappa_{r,\,i}$ is the relative permeability of phase i. Under the assumptions associated with Darcy's law (i.e. low Reynolds number, viscous flow, steady state conditions, homogeneous, isothermal and incompressible fluid) the relative permeability $\kappa_{r,\,i}$ is taken to be only a function of the phase saturation S_i [1]. The existence of the viscous coupling effect [2], which represents the momentum transfer between the two phases, makes this simple extension of Darcy's law highly questionable. Several theoretical, experimental and numerical studies have revealed that two-phase transport in porous media depends strongly on the interfacial morphology and fluid dynamics near the interface. Hence the relative permeability for every phase does not only depend on S_i, but it is also a function of the pore geometry properties, capillary number Ca, wetting angle, viscosity ratio $M = \mu_{nw}/\mu_w$ and flow process (imbibition or drainage). In the past decades, several theoretical approaches have

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been developed to describe viscously coupled multiphase flows in porous medium systems. A volume averaging method was applied to Stokes equation to arrive at a modified theory which includes viscous coupling effects between two fluid phases [3–5]. Marle [6] and Kalaydjian [7,8] employed an approach based on averaging and non-equilibrium thermodynamics to develop analogous transport equations describing the immiscible two-phase flow in isotropic media. Recently, the thermodynamically constrained averaging theory (TCAT) was proposed for modeling multiphase flows by Gray and Miller [9–11]. The TCAT approach is based on the work of Hassanizadeh and Gray [12], Bowen [13], Klaydjian [7] and the importance of fluid–fluid interfaces in multiphase systems has been distinguished and incorporated in model formulations.

Experimental research also attempted to quantify the interfacial coupling effect in the two-phase flow in porous media. Avraam and Payatakes [14–16] performed a series of experiments on a two-dimensional network of pore chambers and throats etched into glass and explored the functional dependence of the relative permeability on the capillary number, wettability, viscosity ratio, and the ratio of injection flow rates. It has been revealed that the steady-state water and oil relative permeabilities may differ substantially from the transient ones depending on the capillary number and wettability by recent experimental research [17]. The dimensionless parameters such as the capillary number and viscosity ratio can influence the capillary pressure and the relative permeability functions [18,19]. Tsakiroglou et al. [18,19], and Aggelopoulos and Tsakiroglou [20] presented the effect of pore space morphology on the transport properties and the capillary pressure-relative permeability relationship. It has been evident that non-random heterogeneities affect strongly the transient flow pattern and the shape of capillary pressure and relative permeability curves.

In recent years, numerous investigations have shown that lattice Boltzmann models are capable of simulating multiphase flows in complex porous systems [21,22]. One of the advantages of the lattice Boltzmann model is that it can solve equations in an arbitrary pore space geometry and topology rather than in a simplification of the pore space geometry [23]. Gunstensen and Rothman [24] investigated the flow of two immiscible fluids in two different three-dimensional microscopic models under a range of applied force and non-wetting fluid saturation by using the lattice Boltzmann model. The results implicated that the generalised two-phase flow model may be a good model for describing the fluid flow when the flow rate is high. A multi-relaxation time (MRT) approximation has also been adopted to evaluated viscous coupling effects in immiscible two-phase flow system [25]. It was found that there is a strong correlation between the relative permeability and capillary number, wettability, the fluid viscosities, and interfacial area between the fluids. Yiotis et al. [26] applied the He-Shan-Doolen LB model to study the immiscible two-phase flow in porous media and reported that the relative permeability of the non-wetting phase may take values greater than unity due to the "lubricating" effect [27] of the wetting films that cover the solid walls. The high-density-ratio gas–liquid flow relative permeabilities change with the wetting saturation S_w, capillary number Ca, and viscous ratio M in heterogeneous porous media has also been studied by using the LB model [28]. Dou and Zhou [29] applied a LB model to investigate the effects of capillary number and viscosity ratio on the non-uniqueness of the relationship between the relative permeabilities and saturation of phases for immiscible two-phase flow in the homogeneous and heterogeneous porous media. Yiotis et al. [30] studied the dynamics of non-wetting liquid blobs during the immiscible two-phase flow in stochastically reconstructed porous media by applying an immiscible LB model. Lehmann et al. [31] used a LB model to investigate the impact of the geometrical properties on the permeability and fluid phase distribution in porous media. The effect of volume, surface, curvature and connectivity (the four Minkowski functionals) on the hydraulic conductivity and the water retention curve was studied. It was found that the volume and surface had a greater impact on permeability of the porous medium than the curvature and connectivity even when the latter ones were altered by 50% or more. However, the authors arrived to the conclusion that a complete description of the porous medium should not be only dependent on the Minkowski functionals but should also be accompanied by the characterisation of the pore sizes in the structure.

All of the previously mentioned studies have revealed remarkable results for immiscible two-phase flows in porous media. However, to the best of our knowledge, there are no systematic LB numerical studies presented in the literature that address the relationship between the geometrical properties and the relative permeabilities for immiscible two-phase flows in pore-scale porous media. The aim of the current paper is to apply a two-dimensional high density ratio MRT LBM for the investigation of the impact of geometrical properties on the immiscible two-phase flows in porous media. The two-dimensional porous structures with different geometrical properties are artificially generated by a Boolean model based on a random distribution of overlapping ellipses/circles. The size and shape of the ellipses/circles are chosen to satisfy the specific Minkowski functionals in two dimensions (i.e. volume fraction, solid line contour length, connectivity). Due to the limitation of computational resources, the current study is restricted to 2-dimensions. This restriction reduces the characterisation of the porous medium to 3 Minkowski functionals as opposed to a three dimensional structure where the effect of curvature (the fourth Minkowski functional) should also be considered. Firstly, the capability and accuracy of the high density ratio MRT LBM is evaluated by simulating the immiscible two-phase co-current flow between two parallel plates. Then, the immiscible two-phase flow in porous media with different non-wetting phase saturation depending on the porosity, contour length and connectivity of solid phase in porous media (i.e. the three Minkowski functionals) is addressed.

2. Models of the pore space

It has been demonstrated that the permeability of sedimentary rock samples is very close to those of Boolean models [32]. Integral geometry furnishes a suitable family of morphological descriptors, known as Minkowski functionals [33,34]. Minkowski functionals are defined for sets Ξ which can be written as finite unions of compact convex sets Ξ_i, $\Xi = \bigcup_{i=1}^{n} \Xi_i$. In two dimensions there are three Minkowski functionals, namely the area A, the contour length C and the Euler characteristic χ which is the number of connected objects minus the number of enclosed cavities. The Boolean model is a certain random set $Z \subset \mathbb{R}^d$ which is used to model various random geometric patterns. The Boolean process can be realised in two simple steps: Firstly, points x_i (called germs) are scattered randomly and independent of each other across space according to a stationary Poisson process. Secondly, a random object Z_i which is called a grain is placed in every random point x_i. An outcome of the Boolean model is then generated as the union set of all such germ-grain pairs.

$$Z = \bigcup_{i=1}^{\infty} (Z_i + x_i). \tag{2}$$

We consider a two-component medium filling a square area $S = l^2$. The global morphology of this two-phase complex material is defined by the global Minkowski functionals per unit area, namely: the solid phase fraction ϕ, the contour length to area ratio L and the mean value of the Euler characteristic $\bar{\chi}$. The global morphology can be related to a Boolean process defined by the