Advances in Water Resources 95 (2016) 80–91

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

Investigating Darcy-scale assumptions by means of a multiphysics algorithm

Pavel Tomin*, Ivan Lunati

Institute of Earth Sciences, Faculty of Geosciences and Environment, University of Lausanne, Mouline - Géopolis, 1015 Lausanne, Switzerland

ARTICLE INFO

Article history:
Available online 21 December 2015

Keywords:
Darcy's law
Multiphysics
Hybrid
Numerical averaging
Macroscopic models
Multiphase flow

ABSTRACT

Multiphysics (or hybrid) algorithms, which couple Darcy and pore-scale descriptions of flow through porous media in a single numerical framework, are usually employed to decrease the computational cost of full pore-scale simulations or to increase the accuracy of pure Darcy-scale simulations when a simple macroscopic description breaks down. Despite the massive increase in available computational power, the application of these techniques remains limited to core-size problems and upscaling remains crucial for practical large-scale applications. In this context, the Hybrid Multiscale Finite Volume (HMsFV) method, which constructs the macroscopic (Darcy-scale) problem directly by numerical averaging of pore-scale flow, offers not only a flexible framework to efficiently deal with multiphysics problems, but also a tool to investigate the assumptions used to derive macroscopic models and to better understand the relationship between pore-scale quantities and the corresponding macroscale variables. Indeed, by direct comparison of the multiphysics solution with a reference pore-scale simulation, we can assess the validity of the closure assumptions inherent to the multiphysics algorithm and infer the consequences for macroscopic models at the Darcy scale. We show that the definition of the scale ratio based on the geometric properties of the porous medium is well justified only for single-phase flow, whereas in case of unstable multiphase flow the nonlinear interplay between different forces creates complex fluid patterns characterized by new spatial scales, which emerge dynamically and weaken the scale-separation assumption. In general, the multiphysics solution proves very robust even when the characteristic size of the fluid-distribution patterns is comparable with the observation length, provided that all relevant physical processes affecting the fluid distribution are considered. This suggests that macroscopic constitutive relationships (e.g., the relative permeability) should account for the fact that they depend not only on the saturation but also on the actual characteristics of the fluid distribution.

1. Introduction

Flow and transport through porous media are determined by the structure of the solid matrix, which is usually assumed inaccessible to the fluids and defines a highly discontinuous and complex pore geometry. Although the physical processes that occur in the pore space are determined by the sub-pore dynamics, the only practicable way to address large-scale problems is to employ macroscopic effective continuum approaches (or simply continuum approaches), which describe the evolution of quantities that are defined by averaging over control volumes of an appropriate size. These approaches have proven effective to model many practical problems as encountered in groundwater and energy applications.

A familiar example is the use of Darcy's law to relate the macroscopic volumetric fluxes (per unit cross area) to the macroscopic pressure gradient. This law (experimentally discovered by Henry Darcy [10] for one-dimensional single-phase flow with no internal source) can be interpreted as a simplistic momentum balance that assumes separation of spatiotemporal scales (i.e., negligible inertia, hence short relaxation time, and control volumes that are large with respect to the relevant pore-scale lengths). Indicating by a the characteristic microscopic length (i.e., the characteristic length scale of the pore-scale process), and by H the observation length (i.e., the typical size of the macroscopic control volume), Darcy's law is expected accurate if $a/H \ll 1$. Under these conditions (and considered that space and time scale are intimately coupled – see, e.g., [33,51]) the drag exerted by the porous matrix within the control volume dominates the viscous fluid–fluid interaction on the external boundary of the control volume and the momentum equation reduces to Darcy's law.

* Corresponding author. Tel.: +16507230611.
E-mail addresses: ptomin@stanford.edu, ptomin@gmail.com (P. Tomin), ivan.lunati@unil.ch (I. Lunati).

http://dx.doi.org/10.1016/j.advwatres.2015.12.013
0309-1708/

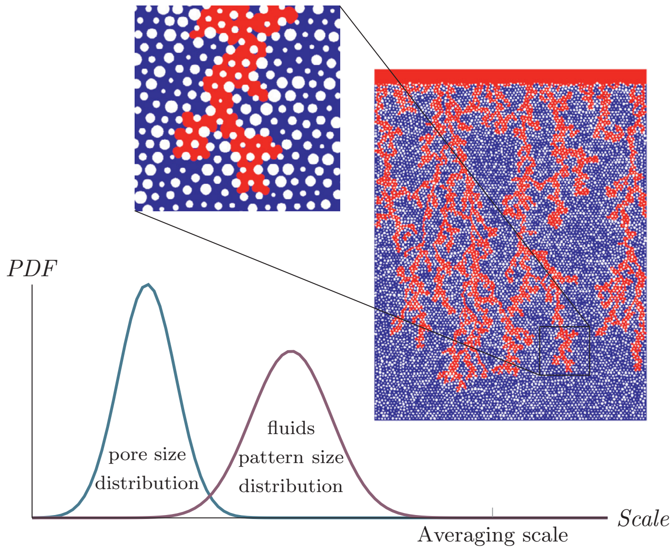

Fig. 1. Schematic representation of distribution of spatial scales for multiphase flow through porous media.

In practice, the observation length is generally large compared to the pore size, and scale separation is satisfied in the case of simple (linear) flow phenomena (e.g., transport of an ideal tracer). However, in presence of complex flow, as it is the case when flow and transport are fully coupled, the processes involved are highly nonlinear and unstable regimes might appear [see, e.g., 14,26–28,31,32,39,43]. The system organizes itself into velocity and fluid-distribution patterns that may have a typical size much larger than the original microscopic length scale. When the microscopic and the macroscopic length scales are comparable ($a/H \sim 1$), Darcy's law may become inaccurate. (See Fig. 1 for a schematic illustration of length scales, and [14,15] for examples of drainage patterns under different flow regimes).

A crude strategy to deal with the lack of scale separation is to resolve the smallest relevant spatial scale. However, this might require solving the Navier–Stokes equations in the pore space to faithfully describe the complex interplay of forces that determines the velocity and the fluid-distribution patterns. Despite the continuous increase in available computational power, full-resolution direct pore-scale simulations of realistic-size problems will remain intractable [14,40,47,55]. To bypass the computational bottleneck, multiphysics (or hybrid) methods can be employed. They combine pore-scale and Darcy-scale models in a single framework, thus allowing a reduction of the computational cost by adopting a pore-scale (or sub-pore scale) resolution only in small portions of the computational domain, whereas a macroscopic description can be employed elsewhere. In general, this methods are applied to single-phase problems, such as reactive transport [6,46,48] or inertial flow regimes [4].

Applying this strategy to multiphase flow problems is substantially more difficult owing to the immiscibility of the phases, which leads to a fluid distribution that is described by a continuum variable at the macroscale (the saturation), but by a discontinuous function at the pore scale (necessary to describe the evolution of the invading front), which requires the solution of highly nonlinear moving interface problems. Recently, we have proposed a multiphysics method for multiphase flow (the Hybrid Multiscale Finite Volume, HMsFV, method [49]), which provides a very general framework to couple different scales and physical descriptions [50], it is adaptive in space and time (see [51] for a detailed discussion of spatiotemporal adaptive simulations of drainage-imbibition cycles), and can be applied to virtually any problem involving multiple physical description (including, e.g., reactive transport, inertial flow regimes, and geomechanical coupling).

The HMsFV method partitions the domain into a set of non-overlapping subdomains (coarse cells) in which local pore-scale problems are solved. These local problems communicate through a global Darcy-scale problem, which provides the boundary conditions. The global problem is constructed by numerical averaging of the microscale equations over the coarse cells (which serve as macroscale control volumes). The accuracy of the multiphysics solutions (with respect to brute-force pore-scale simulations) depends on the ability of the global problem to correctly transfer information between the local problems. As the construction of the global problem is a numerical analog of the conceptual derivation of the macroscopic equations, the method can be used not only as a tool to improve computational efficiency, but also to advance our understanding of the effects of pore-scale processes at larger scales and to investigate the assumptions intrinsic to the macroscopic equations.

In the following, we first describe the HMsFV method focusing on the features that make it an interesting tool to study macroscale models (Section 2). Then, we present several numerical tests that aim at investigating the effects of scale separation and of pore-scale physics on the quality of the multiphysics solution (Section 3). Finally, we conclude by discussing what can be learned from the numerical results about the validity of the Darcy-scale assumptions used to derive the continuum models (Section 4).

2. Multiscale finite volume method

In the last decade, several multiscale methods have been introduced to incorporate the description of different relevant scales into a single framework [1,2,12,13]. All these methods aim at reducing the computational costs of solving large elliptic problems by subdividing the original problem into a set of local fine-scale problems coupled through a global coarse problem. As the global problem is constructed by a numerical upscaling procedure, these methods allow an adaptive update of the coarse-scale parameters and permit a more accurate representation of coarse-scale effects of fine-scale dynamics.

Among these methods, the Multiscale Finite Volume (MsFV) method [24] has the peculiar characteristic of building the global problem by averaging the fine-scale conservation equations over auxiliary coarse-scale control volumes. This procedure closely mimics averaging approaches that are used to derive the macroscopic models. By construction, it ensures the consistency between coarse- and fine-scale fluxes and allows recovering an approximate but fully conservative fine-scale solution by solving a second set of local problems [34,36]. The MsFV method has been extended and applied to a variety of complex problems [9,17–20,25,30,34,35,38,44,54], but it remained essentially a method to efficiently solve the pressure equation that expresses the conservation of total mass.

Recently, we have presented a Hybrid Multiscale Finite Volume method that couples a pore-scale description at the fine scale with a Darcy-like description at the coarse scale [49], and proposed a reformulation of the method that is adaptive in space and time and can be regarded as a general framework to couple different scales and physical descriptions [50,51]. This approach offers the following advantages:

- it is easy to employ different physical descriptions (or different numerical schemes) at the coarse and fine scales (or in different regions of the domain, i.e., for different local fine-scale problems);