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On the methods for determining the transverse dispersion coefficient in river mixing

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ABSTRACT

In this study, the strengths and weaknesses of existing methods for determining the dispersion coefficient in the two-dimensional river mixing model were assessed based on hydraulic and tracer data sets acquired from experiments conducted on either laboratory channels or natural rivers. From the results of this study, it can be concluded that, when the longitudinal dispersion coefficient as well as the transverse dispersion coefficients must be determined in the transient concentration situation, the two-dimensional routing procedures, 2D RP and 2D STRP, can be employed to calculate dispersion coefficients among the observation methods. For the steady concentration situation, the STRP can be applied to calculate the transverse dispersion coefficient. When the tracer data are not available, either theoretical or empirical equations by the estimation method can be used to calculate the dispersion coefficient using the geometric and hydraulic data sets. Application of the theoretical and empirical equations to the laboratory channel showed that equations by Baek and Seo [[3], 2011] predicted reasonable values while equations by Fischer [23] and Boxwall and Guymer (2003) overestimated by factors of ten to one hundred. Among existing empirical equations, those by Jeon et al. [28] and Baek and Seo [6] gave the agreeable values of the transverse dispersion coefficient for most cases of natural rivers. Further, the theoretical equation by Baek and Seo [5] has the potential to be broadly applied to both laboratory and natural channels.

1. Introduction

The mixing mechanism of conservative pollutants in natural rivers is complex due to non-uniformities of the velocity and irregularities of river bed, growth of secondary currents, channel meandering, and so on. River mixing process can be analyzed precisely using a sophisticated three-dimensional model, but this requires considerable effort and time. Since the vertical mixing along the river depth occurs more rapidly than the longitudinal and transverse mixing in most rivers, a two-dimensional model is sufficient to analyze effectively the mixing process. In a two-dimensional mixing model which is derived by averaging the three-dimensional equation over water depth, the three-dimensional effects such as the secondary currents on the mixing process cannot be reflected in the advection terms. The dispersion terms, especially the transverse dispersion coefficient, should contain the three-dimensional effects [39]. Dispersion parameters, i.e., the longitudinal and transverse dispersion coefficients, can be inversely calculated from a numerical simulation model by virtue of iterative techniques until the numerical model results are in good agreement with those obtained from dye tests. Inversely calculated dispersion coefficients, however, are often unrealistic and cannot be applicable to other specific problems because many numerical models usually suffer from approximation errors such as numerical diffusion [32]. Hence many researchers have proposed reliable and independent methods that take into account three-dimensional effects, and facilitate direct determination of dispersion coefficients to avoid the uncertainty related to simulation and to reduce the computational and fiscal burden associated with numerical models.

Methodologies for determination of the transverse dispersion coefficient may be divided into two categories (Fig. 1)—observation and estimation methods. The former method calculates dispersion coefficients using the observed concentration data, which are usually acquired from tracer tests. The latter method estimates dispersion coefficients using basic hydraulic and geometric data in the absence of concentration curves [2]. For observing methods, the moment-based methods suggested by Sayre and Chang [36], Holley [26], Holley et al. [27], Beltaos [9], Krishnappan and Lau [30], Beltaos [10], and so on have been used widely. These have

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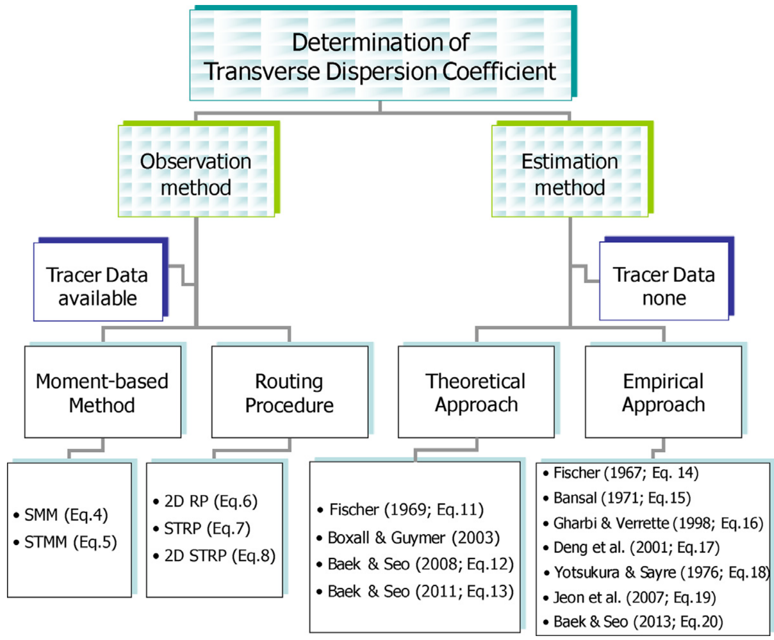

Fig. 1. Methodologies for determining transverse dispersion coefficient (modified from [6]).

some disadvantages that the skewed concentration profile induced by river irregularities makes calculating a meaningful value of the variance of the concentration distribution difficult, and application of moment methods to a transient concentration condition problematic. An alternative method of calculation of the observed dispersion coefficient is a routing procedure which was firstly proposed by Fischer [22]. For estimating methods, there are two ways to determine the dispersion coefficient, theoretical and empirical approaches. The theoretical equation for the dispersion coefficient can be derived from the concept of shear flow dispersion, which makes the predominant contribution to transverse mixing [12,23]. In the empirical approach, an empirical equation is usually proposed using a regression technique based on the basic hydraulic and geometric parameters of rivers. Of course, these two approaches are complementary. A theoretical equation with a complicated form can be simplified by the empirical approach, and the functional relationship of the empirical equation can be derived from the theoretical approach instead of the dimensional analysis [6].

In this study, the methods for determination of the dispersion coefficients in two-dimensional river mixing analysis were reviewed and summarized, and existing methods and equations were compared and assessed using data sets from laboratory meandering channels or natural rivers. This study aimed to suggest criteria for selecting the optimum method of determining the transverse dispersion coefficient under different concentration situations.

2. Theoretical methodology

2.1. Observation method

Under the assumption of a conservative pollutant transported in an open channel with steady flow, the two-dimensional advection-dispersion equation is

$$\frac{\partial(hC)}{\partial t} + \frac{\partial(huC)}{\partial x} + \frac{\partial(hvC)}{\partial y} = \frac{\partial}{\partial x}\left(hD_L\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_T\frac{\partial C}{\partial y}\right) \quad (1)$$

where C is the depth-averaged concentration; t is time; x, y are the longitudinal and transverse coordinates, respectively; u, v are the depth-averaged longitudinal and transverse velocities respectively; h is the local water depth; and D_L, D_T are the longitudinal and transverse dispersion coefficients, respectively. In previous studies of moment-based methods, such as Sayre and Chang [36], Holley [26], and Holley et al. [27], the transverse dispersion coefficient was calculated using concentration data obtained from continuous injection tests in which a tracer was introduced into a channel at a constant rate to generate a steady concentration condition. Thus, moment-based methods have been applied merely to steady concentration fields. To expand the application of moment-based methods to time variable concentration conditions, Beltaos [9] proposed a method for converting the governing equation under the time variable condition into that under the steady condition neither increasing the computational effort nor decreasing accuracy. Eq. (1) is integrated with respect to t, assuming u, v, h, and D_L and D_T are independent of t, to give

$$\frac{\partial}{\partial x}(hu\theta) + \frac{\partial}{\partial y}(hv\theta) = \frac{\partial}{\partial x}\left(hD_L\frac{\partial\theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_T\frac{\partial\theta}{\partial y}\right) \quad (2)$$

where θ is defined as the dosage of the tracer, which is given as

$$\theta(x, y) \equiv \int_0^\infty C(x, y, t)dt \quad (3)$$

Note that Eq. (2) is identical to Eq. (1), with the dosage θ used instead of the concentration C in a steady-state condition, without the time variable term.

The simple moment method (SMM) for the dosage equation was derived by Sayre and Chang [36] under the assumption that the transverse velocity and tracer impingement on banks are negligible, and the longitudinal velocity and depth along the reach are constant. The equation is

$$D_T = \frac{U}{2}\frac{\partial}{\partial x}\left(\frac{\int_0^W \theta y^2 dy}{\int_0^W \theta dy}\right) = \frac{U}{2}\frac{\partial \sigma_y^2}{\partial x} \quad (4)$$

where U is the reach-averaged value of the stream-wise velocity; W is the channel width; and σ_y^2 is the variance of the transverse distribution of θ. The stream-tube moment method (STMM), which was developed by Beltaos [10] to overcome the shortcomings of the simple moment method, can be applied to meandering streams