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Porous gravity currents: A survey to determine the joint influence of fluid rheology and variations of medium properties

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ABSTRACT

We develop a model to grasp the combined effect of rheology and spatial stratifications on two-dimensional non-Newtonian gravity-driven flow in porous media. We consider a power-law constitutive equation for the fluid, and a monomial variation of permeability and porosity along the vertical direction (transverse to the flow) or horizontal direction (parallel to the flow). Under these assumptions, similarity solutions are derived in semi-analytical form for thin gravity currents injected into a two-dimensional porous medium and having constant or time-varying volume. The extent and shape of the porous domain affected by the injection is significantly influenced by the interplay of model parameters. These describe the fluid (flow behaviour index n), the spatial heterogeneity (coefficients β, γ, δ, ω for variations of permeability and porosity in the horizontal or vertical direction), and the type of release (volume exponent α). Theoretical results are validated against two sets of experiments with $\alpha = 1$ (constant inflow) conducted with a stratified porous medium (simulated by superimposing layers of glass beads of different diameter) and a Hele-Shaw analogue for power-law fluid flow, respectively. In the latter case, a recently established Hele-Shaw analogy is extended to the variation of properties parallel to the flow direction. Comparison with experimental results shows that the proposed model is able to capture the propagation of the current front and the current profile.

1. Introduction

Non-Newtonian gravity-driven flow in porous media attracts interest in the context of environmental contamination, as rheological complexity occurs both in environmental contaminants and remediation agents. At the same time, heterogeneity in spatial properties affects the propagation of gravity currents in natural porous media. Gravity currents, generated by the injection of a dense fluid into a light ambient fluid, or vice versa, occur in several natural phenomena and industrial processes (Huppert, 2006). In porous media, the propagation of the current is governed by the interplay between viscous and buoyancy forces, with virtually negligible inertial effects, and it is affected by spatial heterogeneity (Tecklenburg et al., 2013). Relevant examples include seawater intrusion into coastal aquifers (Werner et al., 2013), oil recovery (Loubens and Ramakrishnan, 2011), penetration of drilling-fluids into surrounding reservoirs (Li et al., 2005), and carbon dioxide (CO_2) sequestration in deep subsurface formations (Dentz and Tartakovsky, 2009; Nordbotten and Dahle, 2011). The degree of complexity of models describing porous gravity currents hinges on two main distinctions: first between miscible and immiscible currents, and secondly between confined and unconfined domains. Based on these distinctions, available models to describe the current propagation range from relatively simple analytical schemes, incorporating the sharp-interface approximation and neglecting the motion of the ambient fluid, to full numerical two-phase models incorporating return flows and the effects of diffusion, dispersion, and capillarity. Sharp-interface models are applicable before the transition zone between the two fluids grows too large in case of miscible currents (Szulczewski and Juanes, 2013), or when the effects of capillarity are negligible for immiscible currents (Zhao et al., 2014). The motion of the ambient fluid is typically negligible in the unconfined schematization, when the current depth is much less than that of the porous layer; conversely, it becomes important if the thickness of the intruding current is comparable to that of the porous layer (Huppert and Neufeld, 2014).

Here we focus on a sharp interface schematization of the gravity current, and consider the case of two fluids whose difference in density generates a gravity-driven flow over a horizontal, rigid and impermeable bed inside a saturated porous domain. Contamination scenarios produced by the spreading of dense nonaqueous phase

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liquids in groundwater (DNAPL) may fall into this case (Anderson et al., 2010). From the release source, these substances typically migrate downwards in a shallow phreatic aquifer till the pseudo-impermeable bottom is reached. The conceptualization provided here may describe the subsequent migration of the contaminant.

In several groundwater contamination scenarios, fluids spreading into porous systems exhibit non-Newtonian behavior. Such rheological complexity occurs for: i) the propagation of actual contaminants such as wastewater sludge (Mohapatra et al., 2012), oil pollutants (Theodoropoulou et al., 2001), and waste produced by the resource industries (minerals, coal, and sand-mined oil) (Boger, 2009), and ii) the injection of suspensions employed to enhance the efficiency of in-situ remediation (Kananizadeh et al., 2015; Silva et al., 2013; Tosco et al., 2014). The relevance of the rheology on gravity currents in porous domains has been already assessed in previous studies. Di Federico et al. (2012a, 2012b) modeled the reference cases of homogeneous infinite domains having plane or axisymmetric geometry, with the intruding fluid described rheologically by the Ostwald-DeWaele power-law formulation. Their model was corroborated by the experiments of Longo et al. (2013) in radial geometry. Further modeling advancements were obtained by coupling non-Newtonian rheology with vertical permeability variations in radial geometry (Di Federico et al., 2014) and with the effect of channel boundaries (Longo et al., 2015a).

Here we account for the non-Newtonian nature of the intruding fluid and consider at the same time the influence of soil stratification on two-dimensional gravity-driven flow over an horizontal impermeable bottom. An horizontal stratification is modeled with a continuous variation of properties (permeability and porosity) along the vertical direction, in line with the analysis developed by Zheng et al. (2013) for a Newtonian fluid; the case of two discrete layers was considered by Huppert et al. (2013). A vertical stratification, less common but possible in practical applications, is modeled with a continuous variation of permeability and porosity along the horizontal direction (Zheng et al., 2014).

The resulting problem was already analyzed in a general frame by Zel'dovič and Kompaneec (1950), who analyzed the heat equation with heat conductivity and capacity varying as powers of the temperature. A further extensions by Barenblatt (1952), treated the self-similar motions of a compressible fluid in a porous medium for plane, cylindrical and spherical geometry. In a subsequent paper, Barenblatt (1954) further extended the analysis obtaining limiting self-similar solutions of some special equations, including the classical boundary layer equations. These methods were also earlier employed by Staniukovič (1945). Since these early findings, several papers (e.g., Kamin (1976, 1978); Vázquez (2014) amongst numerous others) used the theory developed by Barenblatt. A detailed exposition of the mathematical theory is contained in the book by Vázquez (2007).

The semi-analytical model analyzes the combined influence of rheology and spatial heterogeneity on the propagation of gravity currents in two-dimensional porous domains. Our formulation is aimed at extending previous results describing simplified scenarios with either permeability gradients or non-Newtonian rheology. A self-similar solution is derived in case of both vertical and horizontal stratifications and for instantaneous or continuous injection. An analysis of the conditions on model parameters required to avoid an unphysical or asymptotically invalid result is presented. A Hele-Shaw analogy is also derived for non-Newtonian power-law flow in two-dimensional porous media for both the configurations analyzed. Model predictions are then assessed against three sets of experiments conducted with two different apparata. A Hele-Shaw cell is used to reproduce simultaneous variations of permeability and porosity along the vertical or horizontal direction. A direct simulation of flow in a stratified packing of glass beads is then conducted to simulate a vertical variation of permeability with constant porosity. In all cases an accurate agreement is obtained between theoretical and experimental results. A set of conclusions closes the paper.

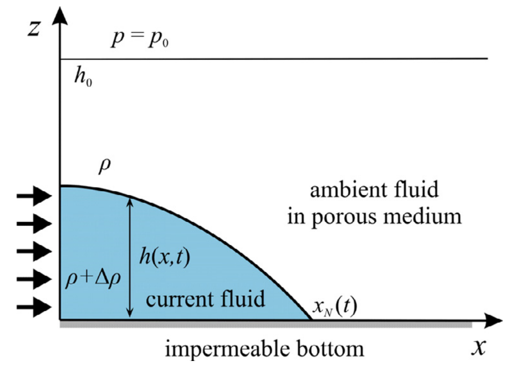

Fig. 1. Sketch of a 2-D gravity current intruding into a saturated porous medium.

2. Problem formulation

The rheological behavior of a non-Newtonian fluid may be described by several models provided in literature (Bird et al., 1960). Among these, the power-law formulation is the simplest one with only two parameters to estimate: the rheological index, n, and the consistency index, $\tilde{\mu}$. In spite of its simplicity, this model holds in a wide range of shear stress/shear rate values (Longo et al., 2013a). The Darcy law for a power-law fluid in a porous medium is given by

$$\nabla p - \rho \mathbf{g} = -\frac{1}{\Lambda k^{(n+1)/2}} |\mathbf{v}|^{n-1} \mathbf{v}. \tag{1}$$

being p the pressure, $\mathbf{g}$ the gravity, $\mathbf{v}$ the Darcy velocity, and k the permeability. In (1) $\Lambda = \Lambda(\phi, \tilde{\mu}, n) = \frac{8^{(n+1)/2}}{2}\left(\frac{n}{3n+1}\right)^n \frac{\phi^{(n-1)/2}}{\tilde{\mu}}$, where ϕ denotes the porosity (Di Federico et al., 2014).

Consider the power-law fluid spreading as a plane gravity current on a horizontal, rigid and impermeable bed into a saturated porous domain (Fig. 1). Motion is driven by the difference in density ($\Delta\rho$) between the lighter fluid, saturating the porous medium, and the heavier intruding fluid. The volume (per unit width) of the intruding fluid is commonly expressed as qt^α, q and α being constants; $\alpha = 0$, and $\alpha = 1$ correspond to constant volume and constant flux injection, respectively.

In a wide range of practical applications it is reasonable to assume: (i) a sharp interface between the two fluids, neglecting mixing, (ii) the intruding current to be thin if compared to its length and to characteristic height h_0 of the porous medium, and (iii) negligible surface tension effects. When these conditions hold, the current spreading may be simply described by its height $h(x, t)$. The ambient fluid is generally taken to be at rest and vertical velocities in the intruding fluid are neglected. Pressure within the current may be approximated by an hydrostatic distribution, expressed for $z \le h$ as $p(x,z,t) = p_0 + \Delta\rho g(h(x,t) - z) + \rho g(h_0 - z)$, where $p_0 = p(z = h_0)$ is a constant. The pressure and the current height are related as $\partial p/\partial x = \Delta\rho g(\partial h/\partial x)$.

The local continuity condition, the global mass balance (accounting for the boundary condition in the origin), and the boundary condition at the current front, $x_N(t)$, complete the formulation:

$$\frac{\partial}{\partial x}\left(\int_0^h u\,dz\right) = -\frac{\partial}{\partial t}\left(\int_0^h \phi\,dz\right), \tag{2}$$

$$\int_0^{x_N(t)} \int_0^h \phi\,dz\,dx = qt^\alpha, \quad h(x_N(t), t) = 0. \tag{3}$$