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Short communication

Quantifying tight-gas sandstone permeability via critical path analysis

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ABSTRACT

Rock permeability has been actively investigated over the past several decades by the geosciences community. However, its accurate estimation still presents significant technical challenges, particularly in spatially complex rocks. In this short communication, we apply critical path analysis (CPA) to estimate permeability in porous rocks from measured mercury intrusion porosimetry and electrical conductivity data. Theoretical estimations of various CPA-based models are then compared to experimental measurements using eighteen tight-gas sandstones. Except for two of the samples, we find permeability estimations performed with the *Skaggs* model (assuming pore diameter independent of its length) more accurate than other models, within a factor of two of the measured permeabilities. We discuss some plausible sources of the uncertainties.

1. Introduction

Accurate estimation of permeability of porous rocks is still of great interest, particularly in multi-phase flow and fluid transport modeling as well as hydrocarbon production. Numerous empirical, semi-physical and physically-based models have been proposed to estimate permeability from other rock properties e.g., porosity (e.g., Kozeny, 1927; Carman, 1937), grain-size distribution (e.g., Morrow et al., 1969; Koltermann and Gorelick, 1995, Porter et al., 2013), mercury intrusion porosimetry (e.g., Katz and Thompson, 1986, 1987), electrical conductivity (e.g., Johnson et al., 1986; Banavar and Johnson, 1987), and nuclear magnetic resonance (e.g., Timur, 1968; Banavar and Schwartz, 1987; Glover et al., 2006).

Among theoretical frameworks developed to model permeability, such as bundle of capillary tubes (e.g., Purcell, 1949; Childs and Collis-George, 1950; Marshall, 1957; Xu and Yu, 2008) and effective-medium approximations (e.g., Doyen, 1988; David et al., 1990; Lock et al., 2004; Ghanbarian and Daigle, 2015), critical path analysis (CPA) from percolation theory (Ambegaokar et al., 1971; Pollak, 1972) appears a reliable method, particularly in heterogeneous and disordered porous media (Liang et al. 2000; Hunt, 2001; Hunt and Gee, 2002; Arns et al. 2005; Bauget et al. 2005a, 2005b; Sahimi, 2011; Hunt et al., 2014). Ambegaokar et al. (1971) argued that fluid flow or transport in disordered media with a broad conductance distribution is dominated by those with magnitudes that are larger than some critical conductance, g_c (corresponding to a critical pore size), which is the smallest conductance among the set of conductances, $g \geq g_c$, forming a sample-spanning cluster. In other words, g_c is the smallest conductance along the path of least resistance through the medium. According to CPA, other finite volume fractions of the porous medium make a negligible contribution to the overall permeability. Therefore, those zones of low permeability may be eliminated from the medium, which would then reduce it to a percolation system (Sahimi, 2011).

In order to model permeability and electrical conductivity in a porous medium, it is necessary to assume specific pore shape and geometrical characteristics. Two common pore shapes presumed in the literature are cylindrical and slit-shaped. The hydraulic (g_h) and electrical (g_e) conductances of a cylindrical pore of diameter d and length l filled with a fluid of viscosity μ and electrical conductivity of σ_w are respectively (Banavar and Johnson, 1987)

$$g_h = \frac{\pi d^4}{128 \mu l} \propto d^{\gamma_h} \tag{1}$$

and,

$$g_e = \frac{\pi \sigma_w d^2}{4l} \propto d^{\gamma_e}. \tag{2}$$

Here $\gamma_h = 4$ and $\gamma_e = 2$, if d and l are independent. If pores in the medium are self-similar, one may assume $d \propto l$, and thus $\gamma_h = 3$ and $\gamma_e = 1$ (Katz and Thompson, 1986; Hunt, 2001; Hunt et al., 2014).

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For slit-shaped pores of width w much narrower than its breadth b and length l Eqs. (1) and (2) change to (Friedman and Seaton, 1998)

$$g_h = \frac{bw^3}{12\mu l} \propto w^{\gamma_h} \quad (3)$$

and,

$$g_e = \frac{\sigma_w bw}{l} \propto w^{\gamma_e} \quad (4)$$

Similarly, $\gamma_h = 3$ and $\gamma_e = 1$ if w is independent of l, and $\gamma_h = 2$ and $\gamma_e = 0$ if $w \propto l$.

Katz and Thompson (1986, 1987) were the first to apply critical path analysis to relate permeability, k, to electrical conductivity and critical pore diameter. They assumed that in the porous medium, cylindrical pore diameter d is linearly proportional to its length l ($d \propto l$) and expressed hydraulic (g_h) and electrical (g_e) conductances as $g_h \propto d^{\gamma_h}$ and $g_e \propto d^{\gamma_e}$ in which $\gamma_h = 3$ and $\gamma_e = 1$. Analogously, one should expect $\gamma_h = 4$ and $\gamma_e = 2$, if d and l are independent, e.g., fixed pore length in a pore-network model.

The Katz and Thompson (1986, 1987) model is given by

$$k = \frac{1}{c}\frac{\sigma_b}{\sigma_w} d_c^2 \quad (5)$$

where σ_b is bulk electrical conductivity, σ_w is saturating fluid electrical conductivity, d_c is critical pore diameter, and c is a constant equal to 226 (hereafter, c_{KT}). Following Chatzis and Dullien (1977) and de Gennes and Guyon (1978), Katz and Thompson (1986) argued that the inflection point on the mercury intrusion porosimetry curve corresponds to the critical pore diameter and the saturation at which sample-spanning cluster first forms. In addition to the mercury intrusion porosimetry, estimating permeability using Eq. (5) requires the electrical conductivity ($\sigma_b/\sigma_w = 1/F$ where F is the formation factor), which may be estimated from mercury intrusion (Katz and Thompson, 1987) or water-expulsion (Nishiyama and Yokoyama, 2014) porosimetry, if not measured.

Eq. (5) is similar in form to the Johnson et al. (1986) permeability model i.e., $k = \Lambda^2/8F$ in which Λ is a characteristic length scale, a measure of dynamically connected pore sizes. Martys and Garboczi (1992) showed that both Λ and d_c are good predictors of permeability k in two-dimensional (2D) pore-network models. Particularly, Martys and Garboczi (1992) stated that, “In a random pore structure, with a distribution of pore sizes, the flow will tend to go more through the largest pore necks, decreasing the importance of the narrowest necks that tend to dominate the behavior of periodic models.” Although Bernabé and Bruderer (1998) documented results similar to Martys and Garboczi (1992) in two dimensions, they found that flow pathways in broadly distributed media were not restricted to the backbone or the critical paths. For permeability modeling using critical path analysis in *highly* heterogeneous and disordered media see Shah and Yortsos (1996).

More recently, Arns et al. (2005) investigated relationships used to estimate permeability from pore size properties in Fontainebleau sandstones based on three-dimensional digitized images. They considered relationships based on the ratio of pore volume to surface area, critical pore diameter (associated with mercury intrusion porosimetry data), characteristic pore sizes associated with nuclear magnetic resonance relaxation time, T_2, as well as mean survival time. Arns et al. (2005) reported that all the investigated relationships provided good agreement with their lattice-Boltzmann simulations. However, permeability values estimated based on critical pore diameter (and critical path analysis) were found to be the most reliable (Arns et al., 2005).

Banavar and Johnson (1987) revisited the Katz and Thompson (1986) model and found that the constant coefficient in Eq. (5) was equal to 7.68×10^{-3} ($c_{BJ} = 130.2$), different from that obtained by Katz and Thompson (1986) ($c_{KT} = 226$). The reason for such discrepancy is that while Banavar and Johnson (1987) maximized the corresponding effective transport coefficient and assumed that the electrical conductivity and/or permeability was proportional to that maximum value, Katz and Thompson (1986) divided that maximum value by the corresponding maximizing pore size (see Banavar and Johnson (1987) for further details).

Following the results of Tyč and Halperin (1989) on random resistor networks with widely distributed conductances, Le Doussal (1989), and more recently Skaggs (2011) proposed the relationship

$$k = \frac{1}{32}\left[\frac{\gamma_h}{\gamma_e}\right]^{-y} \frac{\sigma_b}{\sigma_w} d_c^{\gamma_h - \gamma_e} = \frac{1}{c}\frac{\sigma_b}{\sigma_w} d_c^2 \quad (6)$$

Le Doussal (1989) argued that the prefactor exponent $y = \nu = 0.88$ (ν is the universal correlation length exponent from percolation theory) in three dimensions. However, subsequent numerical simulations of critical path calculation of the conductivity on random resistor networks indicated that $y < \nu$. Skaggs (2003) showed that the observed $y < \nu$ is due to the effects of finite heterogeneity, not finite size, and found $y = 0.74 \pm 0.01$ by means of Monte Carlo simulations.

The values of the Le Doussal (1989) and the Skaggs (2011) constant coefficients (hereafter, c_L and c_S), the numerical prefactor corresponding respectively to $y = 0.88$ and 0.74 in Eq. (6), under different circumstances are given in Table 1. As can be observed, the c_L value differs from c_{KT} by a factor of 3 or 4, depending on the relationship between pore diameter d and its length l. We also list other values of c proposed by Banavar and Johnson (1987) and Friedman and Seaton (1998) in Table 1.

To the best of the authors' knowledge, neither the Le Doussal (1989) nor the Skaggs (2011) model has been evaluated experimentally in porous rocks. Therefore, the main objective of this paper is to compare the permeability estimated from the measured mercury intrusion porosimetry and the electrical conductivity data using CPA-based models e.g., Katz and Thompson (1986), Banavar and Johnson (1987), Le Doussal (1989), and Skaggs (2011) with the measured value.

2. Materials and methods

In order to compare CPA-based models in their estimation of permeability, 18 tight-gas sandstones were selected for the study. Samples were cut from whole core retrieved in a tight-gas sandstone formation located in East Texas. Table 2 summarizes the salient properties of each rock sample. In all samples, permeability was measured by gas flow and corrected (extrapolated to infinite pressure) by the Klinkenberg method (1941). Mercury intrusion porosimetry was used to determine the pore throat-size distribution of each sample. Following Katz and Thompson (1986, 1987), we determined the critical pore diameter from the inflection point on the mercury intrusion porosimetry curve (see Fig. 1 from Katz and Thompson, 1986) assuming that pore shape is cylindrical. For this purpose, we fit a spline to the measured mercury intrusion porosimetry data and numerically calculated the inflection point in MATLAB. Such a method, however, failed to distinguish the critical pore diameter correctly due to local scatter in mercury intrusion porosimetry measurements in 5 samples denoted in Table 2. We show the cumulative pore volume, V, and the difference in pore volume, ΔV, as a function of pore diameter, d, for samples 2 and 18 in Fig. 1. We observe that the ΔV-d relationship for sample 2 is smooth, while it is scattered for sample 18, particularly around the peak, which causes uncertainties in the d_c determination. Following Hofer et al. (2011), in those 5 samples we instead fit the *van Genuchten* capillary pressure curve model (van Genuchten, 1980)