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Saturated, collisional flows of spheres over an inclined, erodible bed between vertical sidewalls

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ABSTRACT

We consider a steady, uniform, dense, fully-saturated, gravity-driven, inclined flow of water and identical spherical grains over an erodible bed between parallel, vertical sidewalls. The grains are inelastic and the energy lost in their interaction is also influenced by the fluid viscosity. We use an extension of kinetic theory for dense flows and employ approximate integrations of the momentum and energy balances for the grains in order to obtain analytical expressions for the depth of flow and for the volume flow rate of the mixture as functions of inclination and average concentration. We also predict the range of slopes for which dense, fully-saturated flows are possible. The predictions are in reasonable agreement with already published experimental results.

1. Introduction

While dry, gravity driven granular flows have been widely analyzed in the scientific literature e.g. [1,2], less attention was devoted to particles–fluid mixtures, which are important in many environmental engineering applications. Debris flows are a representative example, because they involve the flow of highly-concentrated, frequently colliding particles, mixed with water, down steep torrents over an erodible bed, often compromising the safety of people and of their property [3–5]. In professional practice, debris flows are commonly modeled as a single-phase fluid, thus neglecting all the interactions between water and particles, the relation between granular concentration and channel slope and the mechanisms of particle entrainment and deposition. A significant step forward, even if still rarely employed, consists in treating them as a two-phase, fully-saturated flow involving particles and water [6].

The picture presented above motivated us to focus our analysis on flows over an erodible bed between parallel, vertical sidewalls. The depth of the particles and the fluid is the same; that is, the flow is fully-saturated. In what follows, we will introduce balance equations, used to deduce an interval of angles for which dense, collisional flows are possible: for small inclinations, the particles remain at rest and the fluid flows through them. At a critical inclination the particles begin to move. Increasing angles of inclination result in dense flows with increasing depths of particles. Finally, an inclination is reached at which a more dilute, diffuse region developes above the dense flow. In this diffuse, collisional flow, transport of particle fluctuation energy is important and turbulent suspension begins to play a role. Here, we restrict our attention to the dense, fully saturated case, and leave the analysis of the more complicated saturated flows that involve both dense and diffuse regions to a later analysis.

We integrate the mass and momentum balances for the particles and fluid and the balance of fluctuation energy for the particles in an approximate way and obtain analytical descriptions of the evolution of the flow as the inclination is increased. These balances are uniform and independent of time. When treating interactions between pairs of particles, we assume that the interaction involves viscosity [7]. We obtain predictions for the depths and velocities of the saturated flow as a function of the angle of inclination and the particle and fluid properties. The predictions of the theory are discussed and compared with the few experimental results found in literature, testing, in particular, the range of slopes for which fully saturated, dense flows are possible and the relation between the granular concentration and the channel slope.

To the best of our knowledge, two large sets of data for steady, uniform, inclined flows of particle–fluid mixtures have been obtained in laboratory flumes. The depths and volume flow rates of the particles and fluid have been measured as functions of inclination and average particle concentration. The older experiments were performed by Tubino and Lanzoni at the University of Padova [8,9], using three types of sediments: well-sorted gravel with an

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average diameter of 5 and 3 mm, respectively, and glass spheres, with a diameter of 3 mm. More recently, detailed experiments on particle–fluid flows were performed at the University of Trento [10–12]. Although the experimental conditions are relatively simple, with the particle phase consisting of identical PVC cylinders, the measurements are particularly significative in the field of environmental engineering because they are relevant to an understanding of debris flows rheology. The very accurate measurements of velocity, concentration and granular temperature provided evidence for the importance of collisional exchange of momentum and energy between the particles [11,9]. The same experiments have been already analyzed making use of the GDR MiDi constitutive model [13,14]. Here, we employ a more fundamental interpretation based on kinetic theory, extended for dense flows [15,16], and attempt to predict the measurements from a consideration of mass and momentum balances for the particles and the fluid and an energy balance for the particles, introducing some simplifications to obtain analytical expression for the variables of interest.

In what follows, we make lengths dimensionless using the particle diameter d and employ the buoyant gravitational acceleration $\hat{g} = g(1 - 1/\sigma)$, defined in terms of the gravitational acceleration g and the ratio $\sigma = \rho_p/\rho_f$ between the mass density of the particles and that of the interstitial fluid, to make velocities dimensionless using $(\hat{g}d)^{1/2}$ and stresses dimensionless using $\rho_p\hat{g}d$.

2. Balance equations for a steady, uniform, inclined, saturated flow

A first, important difference between dry and submerged particle flows consists in the nature of collisions. Often a coefficient of restitution, ε, is introduced to describe the collision between two spheres, where ε is the negative of the ratio of the components of the particle velocity along the line of centers after and before the collision. The coefficient of restitution is typically measured in dry conditions and is independent of the relative velocity of the two spheres. However the effective coefficient of restitution, e, may reduce dramatically in presence of a fluid, because it is influenced by both the viscosity of the fluid and the strength of the local particle velocity fluctuations. It can be evaluated in terms of the dry coefficient of restitution, ε, and the local Stokes number, St, by [10,17,18]:

$$e = max\left(\varepsilon - 6.9\frac{1+\varepsilon}{St}, 0\right), \tag{1}$$

where $St = \sigma T^{1/2}R/9$, in which T, the dimensionless granular temperature, is one-third the mean of the square of the dimensionless particle velocity fluctuations and $R = \rho_f d(\hat{g}d)^{1/2}/\mu$, with μ the molecular viscosity of the fluid, is the Reynolds number, based on the fall velocity of a single particle. For large values of the Stokes number, the effective and the dry coefficient of restitution may coincide, while e decreases and may even tend to zero, corresponding to the complete dissipation of the collisional kinetic energy, for smaller values of St.

By averaging in the direction normal to the flow and to the sidewalls, the dimensionless particle momentum balances across and along the flow can be written as (e.g., [19])

$$p' = -\nu cos\phi, \tag{2}$$

where the prime denotes the derivative with respect to the coordinate y normal to bed, p is the particle pressure, ν is the particle concentration, assumed to be uniform across the thickness, and ϕ is the angle of inclination of the bed; and

$$s' = -\nu \sin\phi \frac{\sigma}{\sigma - 1} + 2\mu_w \frac{p}{W} - \frac{\nu D}{\sigma}(U - u), \tag{3}$$

where s is the particle shear stress, μ_w is the wall friction, W is the distance between the sidewalls, and D is the drag coefficient:

$$D = \frac{1}{(1-\nu)^{3.1}}\left(0.3|U - u| + \frac{18.3}{R}\right), \tag{4}$$

with U and u the velocity of the fluid and of the particles, respectively. This relatively simple form contains contributions from form drag and viscous drag and is expected to apply over a range of Reynolds numbers from 1 to 10^4.

For dense, collisional flows, the flux of particle fluctuation energy can be neglected in the particle energy balance, which then assumes the simple form [20]

$$0 = su' - \Gamma, \tag{5}$$

where the first term is the rate of production of fluctuation energy and Γ the rate of its dissipation. For dense flows, Γ is expressed as

$$\Gamma = \frac{12}{\pi^{1/2}}\frac{\nu G}{L}(1 - e^2)T^{3/2}, \tag{6}$$

where L is the length of the ephemeral chains of overlapping or chattering particles that develop within the dense flow [15,16], and

$$G = \frac{0.63\nu}{0.60 - \nu} \tag{7}$$

is the product of the granular concentration and the radial distribution function for a contacting pair of spheres for dense flows [21]. Dense regions in fully saturated collisional flows are distinguished from diffuse regions in such flows in that L is greater than unity in the former and equal to unity in the latter.

A balance between the orienting effect of the average shearing and the randomizing effect of the collisions leads to

$$L = \frac{1}{2}\hat{c}G^{1/3}\frac{u'}{T^{1/2}}, \tag{8}$$

where the coefficient $\hat{c}$ is taken to be 0.5, based on the experiments of Pouliquen [22]. Eqs. (5), (6), and (8) can be used to obtain [20]

$$\left(\frac{u'}{T^{1/2}}\right)^3 = \frac{15}{J}\frac{1 - e^2}{\hat{c}G^{1/3}}, \tag{9}$$

where, for very dense flows, J is a function only of the effective coefficient of restitution e:

$$J = \frac{(1+e)}{2} + \frac{\pi}{4}\frac{(3e - 1)(1 + e)^2}{[24 - (1 - e)(11 - e)]}. \tag{10}$$

Eq. (9) can be used to express the dimensionless chain length of Eq. (8) as

$$L = \frac{1}{2}\left[\frac{15}{J}(1 - e^2)\hat{c}^2\right]^{1/3} G^{2/9}, \tag{11}$$

and the particle stress ratio as [20]

$$\frac{s}{p} = \left[\frac{192}{25\pi^{3/2}}\frac{J^2(1 - e)}{\hat{c}(1 + e)^2}\right]^{1/3}\frac{1}{G^{1/9}}. \tag{12}$$

Eqs. (11) and (12) will be employed to limit the extent of the dense, dissipative, fully saturated flow on which we focus.

Finally, the fluid longitudinal momentum balance is

$$S' = -\frac{1 - \nu}{\sigma - 1}sin\phi + \frac{\nu D}{\sigma}(U - u), \tag{13}$$

where S is the shear stress in the water. Eq. (13) is again obtained by averaging in the direction normal to the flow and to the sidewalls. The effect of the fluid is assumed to be restricted to the drag force on the grains and to the reduction of the coefficient of restitution;

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