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Vortex-induced cross-flow seiching in cylinder arrays

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ABSTRACT

An array of vertical cylinders in an otherwise uniform open channel flow can generate a seiche in the transverse direction due to the lift associated with vortex shedding behind each cylinder. Seiche amplitude attains a maximum at resonant conditions, i.e., when the vortex shedding frequency is close to the natural standing wave frequency. In this work we propose a model to predict maximum seiche amplitude based on the analytical solution to the linearized shallow water equations. The model prediction compares favorably with available experimental data from the literature outperforming pre-existing models. The model is also used to highlight that even small disturbances in the flow largely affect the dynamics and can dramatically reduce the wave amplitude.

1. Introduction

When an array of cylinders is placed in an otherwise uniform open channel flow, and the cylinder Reynolds number is in the range of vortex shedding (i.e., $47 < Re = Ud/\nu < 2 \cdot 10^5$ [1], where U is the bulk flow velocity, d is the cylinder's diameter and ν is the kinematic viscosity) a cross-flow seiche is generated due to the lift force associated with vortex shedding.

This phenomenon is sometimes observed in natural systems, e.g. transverse seiching was observed in a canal produced by vortex shedding behind two sets of bridge piers with wave amplitudes as large as 12.5% of flow depth [2]. Hanko [3] also observed transverse oscillations in a spillway with three openings separated by two piers with semicircular cut. The oscillations could be prevented by tapering the downstream ends of the piers in order to dramatically increase the vortex shedding frequency.

Transverse seiching may also have an ecological impact: small floating objects such as seeds and propagulae, as well as eggs of mosquitos and flies, flowing through emergent vegetation are attracted and eventually captured by plants through the Cheerios effect [4–6]. The seiching flow induced by vortex shedding behind emergent stems, by increasing the drag force, can rip out these particles, thus affecting their fate.

More often, transverse standing waves have been observed in laboratory experiments in which an array of cylinders is used to mimic rigid vegetation (see [7] and references therein).

This transverse oscillation, triggered by the periodic shedding of vortices behind each cylinder in an array, shares substantial similarities with the phenomenon of vortex induced vibration (*VIV*) where it is the cylinder that oscillates in the cross-flow direction. For a review on *VIV* the reader is referred to Williamson and Govardhan [8,9].

Within the *VIV* framework, the most well-known feature of the fluid-cylinders interaction is the primary lock-in regime. This occurs when the frequency of vortex shedding, f_v, is close to the natural (or forced) oscillation frequency of the cylinder, f. In this region, vortex shedding is entrained by the cylinder motion and the vortex shedding frequency changes to match the cylinder oscillation frequency.

The qualitative equivalence between *VIV* and the transverse seiching in open channel flow was observed in the experimental investigations by Oengören and Ziada [10–13] on acoustic resonance in tube bundles. In these studies, which focused on the typical frequencies of the phenomenon, the existence of a lock-in region straddling the resonance condition was confirmed for the case of water waves. In this region, vortex shedding is synchronized with the transverse seiching, i.e., vortices shedding from all cylinders, through a feedback mechanism, become correlated and are in phase with each other and with the seiche (see, e.g. Fig. 26 in [12]); in these conditions the largest amplitude oscillations occur. They also found that the natural oscillation frequency is accurately predicted by the small-wave amplitude theory [10,11]:

$$f = \left[\frac{gn}{4\pi B} \tanh\left(\frac{\pi h_0 n}{B}\right)\right]^{1/2} \tag{1}$$

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where g is gravity, n is the mode of the wave oscillation, B is the channel width, and h_0 the mean flow depth.

Only a few investigations have specifically addressed the issue of transverse seiching produced by an array of vertical obstacles in an open channel flow. A first, systematic work is provided by Zima and Ackermann [7] who investigated the phenomenon experimentally. They found that the greatest wave amplitudes occur when the frequency ratio f_v/f is between 0.7 and 1.3, i.e., close to resonance condition, which theoretically attains at $f_v/f = 1$. Under these conditions wave amplitudes as large as 35% of the mean flow depth were measured. On the other hand, when the frequency ratio was less than 0.5 or greater than 2.0, little or no surface waves could be detected. They confirmed that Eq. (1) accurately predicts the natural oscillation frequency.

Zima and Ackermann [7] also proposed an equation to estimate the maximum amplitude of the seiche, A, for the first mode of oscillation. The equation was derived by imposing equilibrium between the pressure force due to the sloping free surface in the transverse direction and the lift force due to the cylinders. The authors recognize that their model fails to consider wave amplification with time due to resonance. However, in deriving their equation, resonance is partially accounted for, since they assume that the natural oscillation frequency of the system coincides with the Strouhal vortex shedding frequency, f_s.

The extension of the Zima and Ackerman [7] equation to modes greater than one, rearranged and written with present notation, reads

$$\frac{A}{h_0} = 2.255\left(\frac{d}{B}\right) n_c d^2 n \frac{1}{S_t^2} \tag{2}$$

where d is the cylinder diameter, n_c is the number of cylinders per unit area, $S_t = f_s d/U$ is the Strouhal number, with U the bulk flow velocity.

A recent contribution to this issue is provided by Sarkar [14] who performed an experimental investigation for the case of an array of randomly distributed, vertical cylinders. He confirmed the existence of a lock-in region and the reliability of Eq. (1) to predict the natural oscillation frequency [14]. In his experiments, however, the cylinders were submerged and did not pierce the free surface. For this reason, his results are not considered in the present study.

Two other works are available in the literature [15,16] in which extensive experimental investigations are presented and discussed within the framework of dimensional analysis. In both these works simple equations relating the seiche amplitude to a set of dimensionless parameters are proposed and assessed through a linear regression procedure on experimental data.

The equation proposed by Ghomeshi et al. [16], rewritten using present notation, reads

$$\frac{A}{h_0} = \begin{cases} 1.41\left(\frac{d}{B}\right)^{1/3}\left(\frac{d}{T}\right)^{1/6}\sqrt{n_c d^2}\, n^{2/3}\frac{1}{\sqrt{S_t}} & T/d > 5 \\ 0.078\left(\frac{d}{T}\right)^{-5/6}\sqrt{n_c d^2}\, n^{2/3}\frac{1}{\sqrt{S_t}} & T/d < 5 \end{cases} \tag{3}$$

where T is the mean transverse spacing between cylinders.

The equation proposed by Jafari et al. [15], rewritten using present notation, reads

$$\frac{A}{h_0} = 0.835K\left(\frac{d}{B}\right)^{1.16}\left(\frac{d}{T}\right)^{-1.42}\left(n_c d^2\right)^{0.7} n^{0.26}\frac{1}{S_t^{0.53}} \tag{4}$$

with $K = 4.27$ for the in line configuration and $K = 1.47$ for the staggered configuration.

The comparison between the above formulas (2)–(4) shows the rather different, sometimes opposing, influence of the main non-dimensional parameters governing the phenomenon on the maximum relative amplitude of the seiche. The comparison also suggests that the phenomenon at hand, despite its apparent simplicity, is actually very complex.

The regression equations proposed by Ghomeshi et al. [16] do not fit with experimental data collected by Jafari et al. [15] and vice versa (see Fig. 1). Experimental uncertainty in measuring wave amplitudes cannot explain the large scatter of data shown in Fig. 1: the many other causes which are likely responsible for the observed scatter are discussed in Section 3.

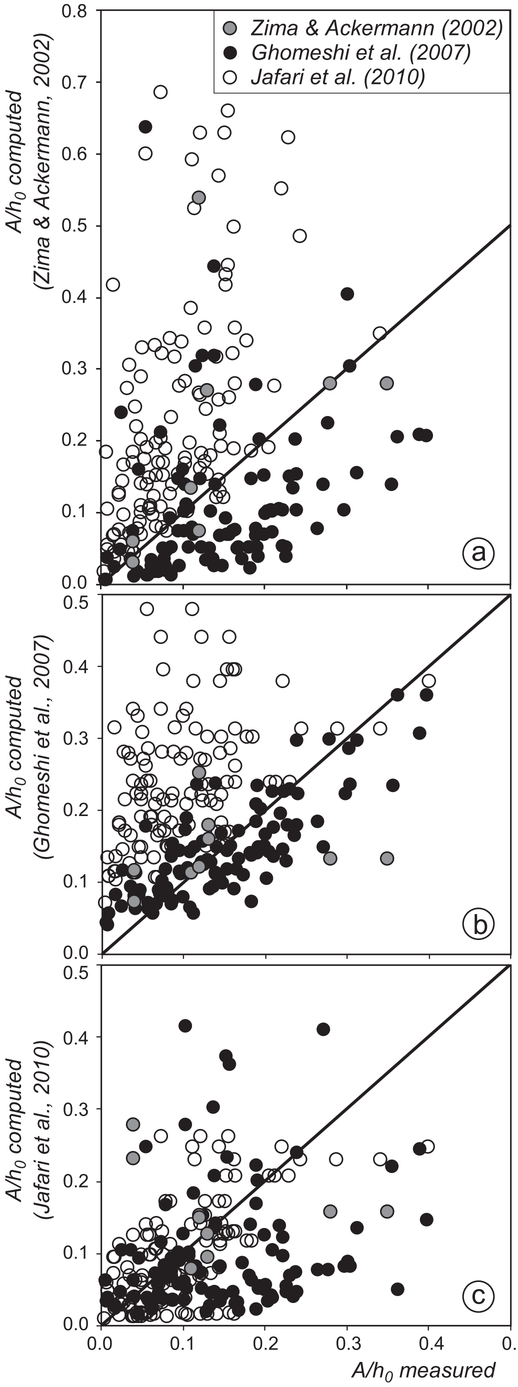

Fig. 1. Comparison between relative amplitude A/h_0 measured by Zima and Ackermann [7] (gray circles), Ghomeshi et al. [16] (black circles), and Jafari et al. [15] (open circles), and the relative amplitude computed with the equations proposed by (a) Zima and Ackermann [7]; (b) Ghomeshi et al. [16]; (c) Jafari et al. [15].