Advances in Water Resources 74 (2014) 127–147

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

An efficient numerical model for multicomponent compressible flow in fractured porous media

Ali Zidane [a], Abbas Firoozabadi [a,b,*]

[a] Reservoir Engineering Research Institute, Palo Alto, CA, USA
[b] Yale University, New Haven, USA

ARTICLE INFO

Article history:
Received 5 March 2014
Received in revised form 12 August 2014
Accepted 18 August 2014
Available online 26 August 2014

Keywords:
Compressible flow
Fractured media
Mixed finite element
Discontinuous Galerkin
Cross flow equilibrium
IMPEC method

ABSTRACT

An efficient and accurate numerical model for multicomponent compressible single-phase flow in fractured media is presented. The discrete-fracture approach is used to model the fractures where the fracture entities are described explicitly in the computational domain. We use the concept of cross flow equilibrium in the fractures. This will allow large matrix elements in the neighborhood of the fractures and considerable speed up of the algorithm. We use an implicit finite volume (FV) scheme to solve the species mass balance equation in the fractures. This step avoids the use of Courant–Freidricks–Levy (CFL) condition and contributes to significant speed up of the code. The hybrid mixed finite element method (MFE) is used to solve for the velocity in both the matrix and the fractures coupled with the discontinuous Galerkin (DG) method to solve the species transport equations in the matrix. Four numerical examples are presented to demonstrate the robustness and efficiency of the proposed model. We show that the combination of the fracture cross-flow equilibrium and the implicit composition calculation in the fractures increase the computational speed 20–130 times in 2D. In 3D, one may expect even a higher computational efficiency.

1. Introduction

Modeling of compositional flow in subsurface fractured media is of interest in geochemical and petroleum reservoir engineering (e.g. gas injection, radioactive waste management). In reservoir simulation, different approximations are used to model flow and transport in the fracture network. Various authors have proposed a large number of models. These models are mainly categorized into two classes: the dual-continuum model and the discrete-fracture/discrete matrix model (DFDM). The most accurate and physically realistic model is the DFDM approach where the fractures and the matrix are both described explicitly in the computational domain [1–9]. In order to capture the matrix/fracture discontinuities in concentration and/or saturation Nick and Matthäi [10] proposed a method in which the mesh at the fracture/matrix interface is split by adding extra nodes/degree of freedom and solve the equations accordingly. This model requires fine grids which makes implementation in field scale fracture network expensive.

The dual-continuum models are widely used to simulate flow in fractured media [11–17] because of low computational cost. The dual-porosity model was first introduced in 1960 [11] and was later in 1963 advanced further [12] to simulate single-phase flow in fractured media. The model was then extended to multiphase flow [13,14,18]. The flow domain is constituted by the connected fracture network, and the matrix domain is constituted by the low permeability rock that provides the storage. In the dual-porosity models, a transfer function that may vary spatially in the domain is used to describe the exchange between the fracture network and the rock matrix. An extension of the dual-porosity model was made by Haggerty and Gorelick [19] employing a so-called multirate mass transfer model at different transfer rates based on the properties of permeable media. Di Donato et al. [20] applied the multiple transfer functions in a single simulation grid block and recently, Geiger et al. [21] extended this model to two-phase incompressible flow. A hybrid model called a fracture-only model was proposed by Unsal et al. [22] for incompressible flow using a dual-porosity approach. A transfer function is used to account for the fracture/matrix exchange, and the fractures are modeled using the discrete-fracture approach. The fracture-only model is based on the assumption that all the fractures are interconnected.

The mass transfer between the fractures and the matrix is described by empirical functions that incorporate some ad-hoc shape factors in all the dual-porosity models. The appropriate shape factors are not well established for compositional and compressible flow. In principle, it is possible to compute these transfer

* Corresponding author at: Reservoir Engineering Research Institute, Palo Alto, CA, USA.

http://dx.doi.org/10.1016/j.advwatres.2014.08.010
0309-1708/

functions to describe fracture-matrix exchange (see e.g. [23]) but as mentioned in [1], there is no theory to calculate the shape factors which determine the exchange between the two domains in two-phase flow with capillary and gravity effects as well as in compositional compressible conditions. In case of diffusion flux, the transfer function approach becomes even less accurate [24].

The control-volume finite element (CVFE) has been used to solve two-phase flow equations in fractured media combined with the Galerkin finite element (GFE) method [25–27]. The fracture entities are embedded within the matrix control-volume in the CVFE method. The calculation of the matrix–fracture flux is therefore avoided and practically there is no difficulty in computing the fracture–fracture flux. However this approach has not been fully examined for compositional modeling and for two-phase flow including gravity and/or capillary pressure effect.

A simplification of the single-porosity model in the discrete fracture approach as proposed in [28,29] assumes that the fracture aperture is small compared to the matrix scale. The fractures are represented by $(n-1)$-dimensional elements in an n-dimensional domain. Hoteit and Firrozabadi [30] used the cross-flow equilibrium (CFE) concept in the discrete fracture approach to model multicomponent compressible flow in fractured porous media. The CFE approach assumes that the pressure in a fracture element is equal to the pressure in the surrounding matrix elements (Fig. 1a and b). This simplification is much more efficient than the single-porosity model and overcomes the limitations of the dual-porosity models. On the other hand, the CFE assumption requires that the matrix grids next to fractures to be small so that the assumption of equality of pressure and composition in the neighborhood matrix elements and fractures is accurate. The size of the time step in the explicit scheme is restricted by the Courant–Freidricks–Levy (CFL) condition. The CFL condition forces the time step to be less than the necessary time for flow to pass through one grid block. The small elements near the fractures impose a severe CFL condition on the time step in the explicit composition calculations. Hoteit and Firoozabadi [31] found that for a fractured reservoir of few kilometers length, the matrix grid-cell size next to the fractures should be in the order of tens of centimeters. A new approach was introduced by Hoteit and Firoozabadi [1] to overcome the limitations of the CFE model and was applied for incompressible two-phase flow in fractured media. In this approach there is a significant increase in computational speed. The essence of the idea by Hoteit and Firoozabadi [1] relates to the constant pressure across the fracture width which is the same as the cross-flow equilibrium across the fracture elements. Note that the cross-flow equilibrium does not imply that convective velocity is zero along the fractures.

In this work we propose a model based on the ideas for incompressible flow suggested by Hoteit and Firoozabadi [1]. There are, however, important differences due to fluid compressibility which requires a pressure equation as we will discuss later. The pressure is assumed to be equal along the fracture width. We refer to the model as the fracture cross flow equilibrium (FCFE). The mass transfer of the species between the fracture and the matrix and inside the fracture network is provided by the hybridized mixed finite element method (MFE). The MFE is more accurate than the traditional finite element (FE) and control volume finite element (CVFE) methods in flux calculation [32,33]. It also has low grid orientation [34]. In addition to the grid-cell pressures, the MFE provides the pressures at the grid-cell interfaces which are used to accurately approximate the velocity field even in highly heterogeneous fractured media [1,30,35–37]. Because of its powerful feature, the MFE is used in this work to discretize Darcy's equation for compressible flow. The species mass balance equation in the matrix is discretized by the discontinuous Galerkin (DG) method [38,39]. The DG method is mass conservative at the element level and has low numerical dispersion. The use of the high-order numerical scheme may produce nonphysical oscillations. To overcome these oscillations, a multidimensional slope limiter is used to

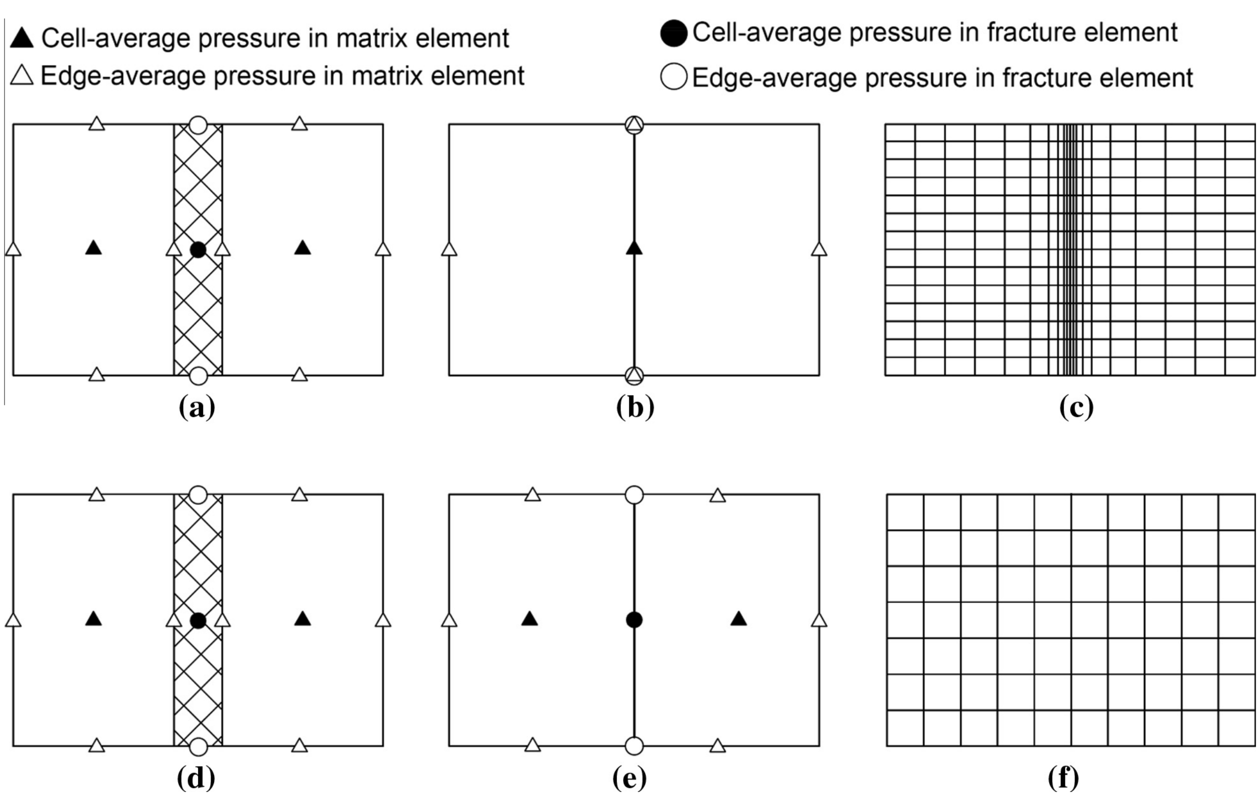

Fig. 1. Physical (a, d) and computational (b, c, e, f) representations in the CFE (a, b, c) and FCFE (d, e, f) approach.