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Series solutions for saturated–unsaturated flow in multi-layer unconfined aquifers

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ABSTRACT

A semi-analytical mesh-free series solution method is presented for modeling regional steady-state subsurface saturated–unsaturated flow in 2-D geometrically complex homogenous and stratified hill-slope cross sections. Continuous solutions for pressure in the saturated and unsaturated zone are determined iteratively, as is the location of the water table surface. Mass balance is satisfied exactly over the entire domain except along boundaries and interfaces between layers, where errors are in an acceptable range. The solutions are derived and demonstrated on multiple test cases. The errors for specific cases are assessed and discussed.

1. Introduction

Modeling of local and regional subsurface flow supports the understanding of regional impacts of human interference and climate change on ground water systems and associated ecosystems [1]. In many cases, both saturated and unsaturated zones must be included in a model to fully describe observed phenomena [2]. In such cases, numerical (rather than analytical) models have generally been used due to the complexity of subsurface flow, which may include non-linear free boundaries, spatially varying material properties with non-linear head dependency, and the presence of geometrically-complex heterogeneity [2–10]. These discrete models are subject to numerical error and their efficiency and accuracy depend upon the resolution of the underlying grid or mesh. Discretization requirements in numerical methods typically contribute to the computational expense of modeling, particularly for free boundary problems where the location of water table is unknown [7,8]. Grid-free (analytical or semi-analytical) methods may be useful for circumventing adaptive mesh and dry cell issues in widely-used numerical software such as MODFLOW [11]. Such models, however, are often limited to geometrically regular and homogenous systems with trivial boundary conditions where the interaction between saturated and unsaturated zones is neglected or overly simplified [12–15]. Recent advances in semi-analytical series solution approaches have relaxed the constraints on geometry by enhancing the traditional method of separation of variables with a simple numerical algorithm [16–18]. The purpose of the research is to extend these semi-analytical series solution approaches for application to regional 2-D steady free boundary saturated–unsaturated subsurface flow induced by spatially variable surface fluxes in geometrically complex homogenous and stratified hillslopes.

2. Background

Semi-analytical methods can be used to accurately tackle complex problems by benefiting from the strength of both analytical and numerical algorithms. For linear or linearized problems, methods such as the series approach used here have the ability to produce continuous and differentiable solutions which satisfy the governing equation(s) exactly. Under many circumstances, they can provide helpful insights into ground water-surface water exchanges in 2-D and 3-D [19]. These powerful tools (e.g., series solutions, separation of variables, Laplace and Hankel transforms, etc.) can be augmented with a simple numerical technique such as weighted least squares minimization (WLS) or numerical inversion to address more geometrically or mathematically complex fully saturated or saturated–unsaturated flow problems [16–24]. Yeh and Chang [24] recently presented a review on analytical and semi-analytical techniques applied to simulate saturated–unsaturated flow in a pumping test. The resulting semi-analytical approaches satisfy mass balance and boundary conditions accurately. To date, researchers have used semi-analytical schemes to independently address the free boundary saturated–unsaturated steady flow in homogenous systems [25] and topography-driven

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saturated flow in heterogeneous aquifers with geometrically complex stratification [18]. However, these issues have never been addressed concurrently. In addition, a robust regional subsurface model requires consideration of the interaction between subsurface flow and the topographic surface. Existing semi-analytical models have paid scarce attention to this issue, and also have neglected the capillary fringe zone [22,23,25]. However, researchers have experimentally and numerically shown that horizontal flow in this zone can have an effect on the magnitude of subsurface flow toward a stream and upon the water table location [2,26].

Multiple researchers have applied semi-analytical series solutions method to simulate topography-controlled saturated steady flow in unconfined aquifers [17,18]. The series solution method is a mesh-free scheme and is able to handle free boundary problems without mesh generation issues. The solutions satisfy continuity of mass exactly over both the saturated and vadose zones. Boundary and continuity conditions along interfaces are satisfied accurately, and explicit and continuous error estimates are immediately available. To simulate the vadose zone using the series solution method, the pressure head-hydraulic conductivity relationship is described using the exponential Gardner model [27]. Pullan [28] has demonstrated that this exponential model is acceptable for a wide range of soils.

3. Problem statement

Fig. 1 shows the general schematic of a stratified soil profile that can be modeled using methods derived herein. An aquifer with length L is subdivided into M layers with arbitrary geometry, each with saturated conductivity K_m^s. Layers are indexed downward from $m = 1$ to $m = M$ and are bounded by the curve $z_m(x)$ above and $z_{m+1}(x)$ below. The bottom bedrock $z_{M+1}(x)$ and sides of the aquifer are impermeable. The topographic surface ($z_1(x)$) is subject to a specified surface flux distribution function (which may be calculated from rainfall, evaporation and transpiration) and/or a Dirichlet condition along surface water bodies (e.g., a river with specified width and head). These conditions are easily amended to account for the presence of multiple surface water features. The saturated–unsaturated interface or top of capillary fringe ($z_{cf}(x)$) is a moving boundary which defines the location of the top of the saturated zone and the bottom of unsaturated zone. The water table is defined as a boundary with zero pressure head. All layer interfaces, the topographic surface, and the bedrock surface are specified prior to solution.

Here the M-Layer system is divided into two zones: the saturated zone (with M_s layers) and unsaturated zone (with M_u layers). The relationship between M, M_s and M_u is a priori unknown and will be discerned through the solution of the problem, since the top of capillary fringe might intersect multiple layers. Note that hereafter (s) and (u) describe saturated and unsaturated properties/variables.

For the saturated zone the problem is posed in terms of a discharge potential, ϕ_m^s [L^2T^{-1}], defined as

$$\phi_m^s = K_m^s h_m(x,z) \tag{1}$$

where $h_m(x,z)$ is the total hydraulic head in layer m, and K_m^s is the saturated hydraulic conductivity of the mth layer [LT^{-1}]. Using continuity of mass and Darcy's law, each saturated layer's discharge potential function must satisfy the Laplace equation:

$$\frac{\partial^2\phi_m^s}{\partial x^2} + \frac{\partial^2\phi_m^s}{\partial z^2} = 0 \quad \text{for } m = M^*,\ldots,M \tag{2}$$

where M^* is the uppermost layer where the top of capillary fringe interface (z_{cf}) exists. For the vadose zone, the problem is expressed in terms of a Kirchhoff potential ϕ_m^u [L^2T^{-1}] in a manner similar to [27,29]. The Kirchhoff potential is a function of pressure head φ [L]

$$\phi_m^u(\varphi_m) = \int_{-\infty}^{\varphi_m} K_m(\eta)d\eta \tag{3}$$

and the gradient of this potential corresponds to the flow rate in the mth unsaturated layer. Using the exponential Gardner model with air entry pressure (as originally included by Rijtema [30])

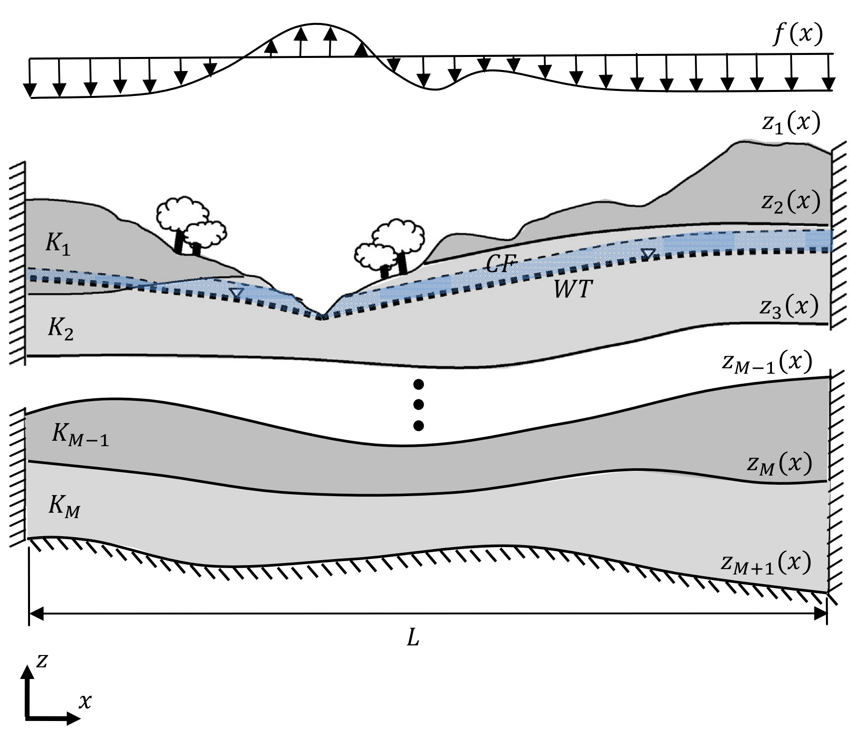

Fig. 1. Layout of the general problem. M layers are separated by the interfaces $z_m(x)$, with $z_{M+1}(x)$ corresponding to the bottom bedrock and $z_1(x)$ corresponding to the topographic surface. CF refers to the boundary between saturated and unsaturated zones.