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Hindered settling with an apparent particle diameter concept

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ABSTRACT

In this work a new expression has been developed to predict the settling velocity of a sediment particle which is dispersed in a sediment-fluid mixture during a turbulent flow. A concept of apparent particle diameter has been introduced and is defined by the diameter of the spherical volume in which the particle can move randomly after collision with other particles in suspension. The effect of suspension concentration is studied on the mass density of the sediment-fluid mixture. It has been shown that the settling velocity of sediment particle in a sediment-fluid mixture is a function of different characteristics of the sediment particle such as settling velocity in clear fluid, suspension concentration, relative mass density and Reynolds number. The model has shown good agreement when compared with previously published experimental data and it's prediction accuracy is superior than the other existing models.

1. Introduction

Settling velocity is one of the important parameters in the study of sediment transport due to its importance in suspension concentration, deposition, exchange process between upward and downward flux in the diffusion process and other various aspects in sedimentation. A large number of expressions of the settling velocity of a sediment particle in clear fluid are available in literature [5,13,29]. Researchers [16,17] observed that in a sediment-fluid mixture due to increased suspension concentration, the settling velocity of a particle is reduced in comparison to the settling velocity in clear fluid and this fact is known as 'hindered settling', in a simpler language which means 'delayed settling' or 'obstructed settling'. From physical point of view, this reason can be explained by different forces acting on the particle such as drag, gravitational, pressure, buoyancy forces etc. But from the view of fluid dynamics, it is needed to modify the flow field of the sediment particle where the settling is happening in the vicinity of other suspending particles. The widely used simple Richardson and Zaki [20] equation is the best in this regard and the hindered settling according to them is

$$\omega_m = \omega(1-c)^{n_H} \tag{1}$$

where ω_m and ω are the settling velocity of the sediment particle in sediment-fluid mixture and clear fluid respectively and n_H is the exponent of reduction of settling velocity in sediment-fluid mixture. They suggested that n_H depends on the particle Reynolds number R, which is $\omega d_p/\nu_f$ where d_p is the particle diameter and ν_f is the kinematic viscosity of clear fluid and proposed the following relations

$$n_H = \begin{cases} 4.65, & R < 0.2, \\ 4.4R^{-0.03}, & 0.2 < R < 1. \\ 4.4R^{-0.1}, & 1 < R < 500. \\ 2.4, & 500 < R. \end{cases} \tag{2}$$

Garside and Al-Dibouni [12] empirically proposed the expression of n_H as

$$\frac{5.1 - n_H}{n_H - 2.7} = 0.1R^{0.9} \tag{3}$$

Thacker and Lavelle [24] analyzed the hindered settling by the two phase flow analysis. Chien and Wan [7] also proposed expression of n_H as a function of R. An equation was developed by Cheng [4] which is a relation between n_H, suspension concentration, particle Reynolds number R and relative mass density s where s is the ratio of mass density of sediment particle ρ_p to that of clear fluid ρ_f. Researchers have done several experiments under various conditions and observed that the experimental settling velocity is lower than the settling velocity predicted by Eq. (1) and they modified n_H to a higher degree of accuracy [19,23]. Baldock et al. [2] experimentally studied the settling velocity at high concentrations for sediment particles with different combination of d_p and s. The effect of the particle shape was included by Tomkins et al. [26] to study the sedimentation rate of the sand grains in the hindered settling regime. Recently, Van and Bang [27] studied the segregation effect between sand grains and mud flocs during hindered settling and their simulations are proposed by using two coupled mass conservation equations.

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Nomenclature

A_1, A_2	constants	R_m	modified Reynolds number of sediment particle
c	volumetric concentration of sediment particle in suspension	s	$(=\rho_p/\rho_f)$ relative mass density of sediment particle
C_D	drag coefficient	v_f	volumetric fraction of clear fluid
C_{D_m}	apparent drag coefficient	v_p	volumetric fraction of suspended particle
c_{max}	maximum volumetric concentration of suspended particle	V_{ci}, V_{oi}	computed and observed values of the ith data point
d_m	apparent sediment particle diameter	Δ_m	$(\rho_p - \rho_m)/\rho_m$
d_p	sediment particle diameter	Δ_p	$(\rho_p - \rho_f)/\rho_f$
D_*	non-dimensional particle diameter	$[\mu]$	intrinsic viscosity
D_{*m}	modified non-dimensional particle diameter	μ_f	dynamic viscosity of clear fluid
E_r	relative error in percent	μ_m	dynamic viscosity of sediment-fluid mixture
f	function of c and $\triangle_p$	μ_r	relative viscosity of sediment-fluid mixture
F_B	buoyancy force on sediment particle	ν_f	kinematic viscosity of clear fluid
F_D	drag force on sediment particle	ν_m	kinematic viscosity of sediment-fluid mixture
F_G	weight of sediment particle	ω	settling velocity of sediment particle in clear fluid
g	gravitational acceleration	ω_m	settling velocity of sediment particle in sediment-fluid mixture
n_H	exponent of reduction of settling velocity	ω_r	$(=u_s - u_f)$ relative velocity between sediment particle and fluid
N	number of data points	ρ_f	mass density of clear fluid
$\vec{u_1}$	velocity vector of solid phase	ρ_p	mass density of sediment particle
$\vec{u_2}$	velocity vector of fluid phase	ρ_m	mass density of sediment-fluid mixture
u_f	vertical suspended fluid velocity		
u_s	vertical sediment particle velocity		
R	Reynolds number of sediment particle		

Though many empirical expressions of n_H are available in literature, but few researchers have studied the theoretical. To presume the settling velocity of a sediment particle in sediment-fluid mixture more accurately, a theoretical study of n_H is carried out in this work to modify Eq. (1). Also the effect of suspension concentration is studied on the mass density of sediment-fluid mixture. This effect together with the effect of kinematic viscosity of sediment-fluid mixture are used to develop a model on n_H incorporating a concept on apparent particle diameter. A wide range of previously published experimental data are used for the verification of the proposed model [2,8,9,11,14,26,28].

2. Mathematical modeling

2.1. Dynamical characteristics of a sediment particle settling in clear fluid

Let c be the volumetric concentration of the sediment in suspension and therefore $1-c$ is the volumetric concentration of the suspended fluid. The continuity equation for the solid phase is

$$\frac{\partial c}{\partial t} + \nabla.(\vec{u_1}c) = 0 \tag{4}$$

and for the fluid phase is

$$\frac{\partial (1-c)}{\partial t} + \nabla.(\vec{u_2}c) = 0 \tag{5}$$

where $\vec{u_1}$ and $\vec{u_2}$ are the velocity vector of the sediment particle and suspended fluid respectively and t is the time. Addition of Eqs. (4) and (5) leads to

$$\nabla.(\vec{u_1}c + \vec{u_2}(1-c)) = 0 \tag{6}$$

Since only the settling velocity of the sediment particle is the matter of concern of the present study, the vertical direction is considered. Also by the definition, settling velocity is a constant velocity i.e. it has no change with time; as such integration of Eq. (6) gives

$$u_s c + u_f(1-c) = c_s = 0 \tag{7}$$

where c_s is the constant of integration, u_s and u_f denote the velocity of the sediment particle and suspended fluid respectively in the vertical direction. The constant term c_s of Eq. (7) is taken as zero due to equilibrium between solid and liquid phases during the motion. The equilibrium can be explained by the fact that when a certain volume of sediment is moving up, same volume of fluid must be moving down and vice versa and as a result no empty space is created in suspension. u_s is considered as the settling velocity ω_m of the sediment particle in the sediment-fluid mixture and from Eq. (7), the relative velocity $\omega_r(= u_s - u_f)$ between the falling of sediment particle and the fluid can be written as

$$\omega_r = \frac{\omega_m}{1-c} \tag{8}$$

When a sediment particle falling downwards reaches the settling velocity, the sum of different forces acting on it must be equal to its weight F_G. In a clear fluid, the total force on a sediment particle can be separated into drag (F_D) and buoyant force (F_B) and for the equilibrium position

$$F_G = F_B + F_D \tag{9}$$

The expression of F_G, F_D and F_B can be written as

$$F_G = \frac{1}{6}\pi d_p^3 \rho_p g \tag{10}$$

$$F_B = \frac{1}{6}\pi d_p^3 \rho_f g \tag{11}$$

$$F_D = \frac{1}{2} C_D \rho_f \omega^2 \frac{\pi d_p^2}{4} \tag{12}$$

where C_D is the dimensionless drag coefficient. Substituting F_G, F_D and F_B in the Eq. (9) one can obtain the expression of C_D as

$$C_D = \frac{4}{3}\frac{\Delta_p g d_p}{\omega^2} \tag{13}$$

where Δ_p is $(\rho_p - \rho_f)/\rho_f$. In general, C_D is a function of the Reynolds number R of sediment particle. For Stokes flow when $R < 1, C_D$ is