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A numerical method for two-phase flow in fractured porous media with non-matching grids

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ABSTRACT

We propose a novel computational method for the efficient simulation of two-phase flow in fractured porous media. Instead of refining the grid to capture the flow along the faults or fractures, we represent the latter as immersed interfaces, using a reduced model for the flow and suitable coupling conditions. We allow for non matching grids between the porous matrix and the fractures to increase the flexibility of the method in realistic cases. We employ the extended finite element method for the Darcy problem and a finite volume method that is able to handle cut cells and matrix-fracture interactions for the saturation equation. Moreover, we address through numerical experiments the problem of the choice of a suitable numerical flux in the case of a discontinuous flux function at the interface between the fracture and the porous matrix. A wrong approximate solution of the Riemann problem can yield unphysical solutions even in simple cases.

1. Introduction

It has been observed that fractures and faults in porous media can act as conduits or barriers for the flow. A relevant application that requires an accurate characterization of the faults from the geo–mechanical and hydrodynamic point of view is the study of CO_2 injection and storage [22]. At injection conditions, CO_2 is buoyant relative to the ambient groundwater, so it rises toward the top of the formation and, in the presence of a pre-existing well or fracture, or the activation of a fault, could leak into shallower formations.

The effect of fractures on the flow is important also in many different applications such as the study of fractured aquifers, geothermal fields, oil and gas reservoirs and unconventional hydrocarbon sources.

In all the aforementioned applications the presence of large fractures or faults influences the flow in a complex way that cannot be reproduced in numerical simulations by simple homogenization. The spatial scale of these features is usually such that a very fine mesh is needed, leading to an extremely high computational cost. The typical width of fractures (of the order of centimetres) and faults (of the orders of meters) is indeed very small compared to the size of the domain of interest that ranges from hundreds of meters for reservoir scale simulations to hundreds of kilometres at basin scale.

One possibility to address this problem is to use a reduced model to account for the flow in fractures, represented as $(n-1)$ dimensional objects coupled with the rest of the porous medium, i.e. to use a discrete fracture model. A reduced model for the flow along the fracture and suitable coupling conditions at the matrix-fracture interface were derived by Alboin et al., [2] for single-phase Darcy problems in porous media in the presence of permeable fractures, and later extended by Martin et al., [23] and Angot, [3] to more general coupling conditions and geometric configurations. Discrete fracture models have been approximated using a variety of numerical methods including finite differences, finite elements and finite volumes. In all the cases above the fractures are usually represented by gridcells interfaces, i.e. the grid is conforming with the fractures. In [9] the authors extended the approach of [23], based on mixed finite elements, to allow for non-matching grids between the porous medium and the fracture thanks to the use of the extended finite element method (XFEM). Removing the constraint of mesh conformity can be convenient in realistic cases with numerous and complex fractures. Another advantage of a non-matching method is the possibility to run multiple simulations with different fracture configurations, without meshing the domain each time. This can be useful in the study of uncertainty on geophysical parameters or multiple scenario analyses.

In this paper we present an original numerical approximation strategy for two-phase flow in fractured media with non-matching grids. We point out that a large variety of method is available in literature for the simulation of multiphase flows with discrete fracture models in the conforming mesh case. Among others, cell centered finite volumes methods have been used in [20] with a

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two-point flux approximation method, while in [17] the authors proposed a method based on mixed finite elements with a formulation that allows to model the saturation discontinuity due to capillary pressure heterogeneities. The purpose of this work, however, is to extend the approach of [9] to develop a numerical method for two-phase flow that can handle non conforming grids. This implies that the fractures are no longer represented as grid cells interfaces, and that the grid of the fracture is completely irrespective of the discretization of the porous matrix. To focus on this new aspects we introduce some simplifications in the model, in particular, we restrict ourselves to the two-dimensional case, consider incompressible flows and neglect capillary effects. We complement the generalized Darcy problem, approximated as in [9], with an evolution equation for the saturation of one of the two phases, to obtain a fractional flow formulation of the two-phase Darcy problem, starting from the solution strategy introduced in [18]. In the absence of capillary pressure we set up for the hyperbolic saturation equation a discretization scheme that can handle non matching grids. In particular, the finite volume scheme should deal with cut cells and account for the interactions between the porous matrix and the fracture. As concerns the time discretization an explicit scheme is considered and the coupling between the Darcy problem and the saturation equation is solved via an Implicit Pressure – Explicit Saturation (IMPES) splitting. The faults and the surrounding porous matrix can be regarded as two different rock types characterized by different values of the absolute permeability tensor. In general, they might also be assigned different relative permeability functions. As a result the saturation equation has a flux function that is discontinuous at the matrix-fracture interface. In this type of problems the choice of the numerical flux is crucial to obtain an accurate solution. It was shown [24] that in some cases an approximate solution of the Riemann problem at the interface could lead not only to inaccurate solutions but, more important, yield unphysical solutions that do not satisfy the entropy condition. In this work we will employ the upstream mobility flux for the solution of some relevant test cases and compare the results with those obtained with an exact Riemann solver to identify possible limits of the approximate solvers for the problem of our interest. The paper is organized as follows. In Section 2 the governing equations for the two-phase flow are presented. The reduced model for the flow in the fracture and the interface conditions that couple the bulk and fracture flow are discussed in Section 3. The numerical discretization of the problem is presented in Section 4. Section 5 is devoted to the solution of some synthetic test cases with the aim of verifying the properties of the method and assessing its effectiveness.

2. Governing equations

We consider two immiscible and incompressible fluid phases, denoted by the superscript $\alpha \in \{w, n\}$ for the wetting and non-wetting phase respectively, flowing in a saturated porous medium crossed by fractures. The latter can be characterized by soil properties, *e.g.* permeabilities, that differ significantly from the porous matrix. We neglect the effect of capillary pressure.

Let us consider a regular domain $\Omega \in \mathbb{R}^n, n = 2$ or 3, with the boundary $\overline{\Gamma} = \overline{\Gamma}_N \cup \overline{\Gamma}_D, \Gamma_N \neq \emptyset$, and an outward unit normal $\boldsymbol{n}_\Gamma$, cut by a thin region $\Omega_f \subset \Omega$ of thickness d representing the fracture. Let us set, from now on, $i \in \{1, 2, f\}$ and $j \in \{1, 2\}$. Fig. 1 represents the partition $\overline{\Omega} = \bigcup_i \overline{\Omega}_i$ into three disjoint subsets of Ω. The interfaces between Ω_j and Ω_f are denoted as $\gamma_j \in \mathbb{R}^{n-1}$ and have a unit normal $\boldsymbol{n}_j$, pointing outwards with respect to Ω_j. If we introduce the interval of time $\mathcal{I}_T := (0, T)$ the space–time domains are defined as $Q_i := \Omega_i \times \mathcal{I}_T$. The equations that describe the two-phase flow can be written in the so called *fractional flow formulation*

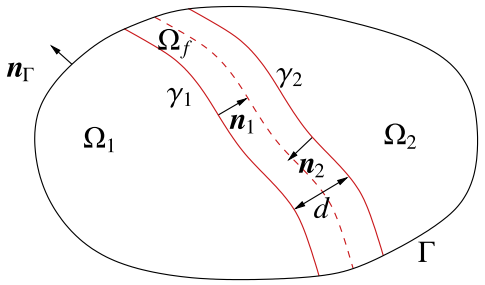

Fig. 1. Sketch of $\Omega \in \mathbb{R}^2$ with Ω_1 and Ω_2 divided by Ω_f.

[7,18], as a system formed by a generalized Darcy problem and the saturation equation. The unknowns are the total velocity $\boldsymbol{u}$ defined as the sum of the two phase velocities, the global pressure p, an artificial variable related to the phase pressures, [7], and the saturation of the non-wetting phase, from now on denoted as S, with $S \in [0, 1]$. We have assumed that $S_w + S_n = 1$, *i.e.* no residual saturation. The system reads, for each subdomain Ω_i, in the dimensional form as

$$\begin{cases} \nabla \cdot \boldsymbol{u}_i = 0 \\ \boldsymbol{u}_i = -\lambda_i \boldsymbol{K}_i (\nabla p_i - \boldsymbol{G}_i) \\ \Phi_i \frac{\partial S_i}{\partial t} + \nabla \cdot \boldsymbol{v}_i = 0 \\ \boldsymbol{v}_i = f_i \boldsymbol{u}_i + b_i \boldsymbol{K}_i \boldsymbol{g} \end{cases} \quad \text{in } Q_i, \tag{1a}$$

with the coupling conditions

$$\begin{cases} \boldsymbol{u}_j \cdot \boldsymbol{n}_j = \boldsymbol{u}_f \cdot \boldsymbol{n}_j \\ p_j = p_f \\ \boldsymbol{v}_j \cdot \boldsymbol{n}_j = \boldsymbol{v}_f \cdot \boldsymbol{n}_j \end{cases} \quad \text{on } \gamma_j \times \mathcal{I}_T, \tag{1b}$$

where the subscripts i and j denotes the restriction of the variables to Q_i or γ_j, respectively. Furthermore we have set

$$\lambda_i := \frac{k_i^n}{\mu^n} + \frac{k_i^w}{\mu^w}, \quad \boldsymbol{G}_i := \frac{k_i^w \rho^w / \mu^w + k_i^n \rho^n / \mu^n}{\lambda_i} \boldsymbol{g},$$
$$f_i := \frac{k_i^n}{\mu^n \lambda_i}, \quad b_i := \frac{k_i^n k_i^w}{\mu^n \mu^w \lambda_i} (\rho^n - \rho^w).$$

Here $\boldsymbol{K}_i$ denotes the absolute permeability tensor which is symmetric and positive definite and Φ_i is the porosity. It is important to note that in realistic situations the entries of $\boldsymbol{K}_i$ can differ in several orders of magnitude from the neighboring subdomains. For each phase α, k^α is the relative permeability, ρ^α the density and μ^α the dynamic viscosity. Finally $\boldsymbol{g}$ is the gravity acceleration vector. The relative permeabilities are non-linear functions of the saturation [4] and in principle different shapes could be associated with different rock types.

Let us introduce a discretization of the time interval, dividing $\mathcal{I}_T$ into N subintervals

$$\mathcal{I}_T^m := (t^m, t^{m+1}) \quad \text{with} \quad \Delta t^m := |\mathcal{I}_T^m|,$$

such that $\overline{\mathcal{I}_T} = \bigcup_m \overline{\mathcal{I}_T^m}$ for $m \in \{0, \ldots, N\}$. Using an *IMPES* type approach [7] to solve (1) we decouple the pressure equations from the saturation equations, and we solve them in sequence at each time step $\mathcal{I}_T^m$. Hence the pressure equations, with fixed saturation $S^{(m)}$ at $\mathcal{I}_T^m$, read

$$\begin{cases} \nabla \cdot \boldsymbol{u}_i = 0 \\ \boldsymbol{u}_i = -\lambda_i(S^{(m)}) \boldsymbol{K}_i \left[\nabla p_i - \boldsymbol{G}_i(S^{(m)}) \right] \end{cases} \quad \text{in } \Omega_i \times \mathcal{I}_T^{m+1}, \tag{2a}$$

with the following coupling conditions

$$\begin{cases} \boldsymbol{u}_j \cdot \boldsymbol{n}_j = \boldsymbol{u}_f \cdot \boldsymbol{n}_j \\ p_j = p_f \end{cases} \quad \text{on } \gamma_j \times \mathcal{I}_T^{m+1}. \tag{2b}$$