Estuarine, Coastal and Shelf Science 183 (2016) 163–178

Contents lists available at ScienceDirect

Estuarine, Coastal and Shelf Science

journal homepage: www.elsevier.com/locate/ecss

The effect of density gradient on boundary flow

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ARTICLE INFO

Article history:
Received 29 February 2016
Received in revised form
12 October 2016
Accepted 18 October 2016
Available online 19 October 2016

Keywords:
Velocity profiles
Turbulent shearing stress
Eddy viscosity
Density gradient
Boundary layer

ABSTRACT

Based on Prandtl's mixing-length theory, this study proposes new theoretical formulae of velocity profiles resulting from the forcing by a constant horizontal buoyancy gradient in unstratified and stably stratified flows. Based on the one-dimensional water column momentum equation, the vertical turbulent shearing stress profile is found to deviate from a linear distribution and follow a parabolic distribution, differing from that in neutral flow. The shearing stress curves upward with the current following the density gradient, and curves downward with the currents opposite to the density gradient. For a constant eddy viscosity, the well-known estuarine circulation is obtained through the parabolic shearing distribution. For a vertically parabolic eddy viscosity, the new-proposed velocity profile by Burchard & Hetland (2010) is obtained. In this paper, we estimate the viscosity profile based on Prandtl's mixing-length theory and then derive the new formulae of the velocity profiles. Through comparison with the numerical turbulence model, the velocity profiles presented in the paper agree well with the numerical results. In addition, the new velocity profiles can be applied to determine the valid range and evaluate errors of the log-fit in baroclinic boundary flows.

1. Introduction

In estuarine and coastal areas, numerous velocity profiles through the bottom boundary layer (BBL) indicate a deviation from the classic logarithmic depth dependence. Some investigators attributed the deviation to several factors, such as acceleration and deceleration of tidal currents, density stratification, the transport of material as bed-load, etc. (Collins et al., 1998). Among these factors, the effects of density gradient, including longitudinal density gradients and vertical stratification on the velocity profiles, have been recognized by many investigators as a significant factor in the action of current circulation and salt water intrusion in the field of estuarine and coastal physical oceanography (Soulsby and Wainwright, 1987; Hetland and Geyer, 2004).

Early studies of the well-mixed estuaries assumed that the flow was influenced persistently by the horizontal density gradient, namely the baroclinic effect, which brought the near-bed seawater landward and counteracted some near-bed runoff seaward. The well-known estuarine circulation theory was proposed and analyzed by Pritchard (1952, 1954), Hansen and Rattray (1965), and Chatwin (1976), with the assumption that the geometry was idealized and the mixing was constant throughout the estuary. The formula of estuarine circulation was later used widely to interpret observations in the well mixed and partially mixed estuaries by many scholars (Geyer, 1997; Geyer et al., 2000; Vinita et al., 2015). Their theory assumed that, if the section averaged salinity increased linearly along the estuary, then both the gravitational and the stratification circulation were constant along the estuary. Prandle (2004) established theoretical solutions for the vertical profiles of tidal currents and residual currents under a linear approximation to the familiar quadratic bed friction law. His results indicated that neglecting the longitudinal density gradient was entirely valid for vertical structure of tidal currents but incorrect for that of the residual current because the density gradient introduced small, but significant, tidally averaged residual currents. Different from Hansen and Rattray (1965) and Prandle (2004), who assumed a constant eddy viscosity, Burchard and Hetland, (2010) presented a log-linear expression for tidal-mean currents, assuming a more realistic parabolic eddy viscosity, showing that the intensity of the

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http://dx.doi.org/10.1016/j.ecss.2016.10.025
0272-7714/

gravitational circulation scaled with the horizontal Richardson number. MacCready (2004) concluded that all these theories relied on knowledge of the vertical turbulent momentum flux and the Hansen and Rattray (1965) solution always became diffusion-dominated near the mouth. Recent studies concentrated on internal tidal asymmetry (called tidal straining), which was thought to be a major mechanism to form estuarine circulation (Simpson et al., 2002, 2005).

On the other hand, the investigation into the effects of density stratification on the velocity profiles was initiated by scholars and engineers outside the field of estuarine and coastal physical oceanography. Numerous experimental results from atmospheric fluid dynamics introduced the Monin-Obukov length to account for the bottom boundary layer under the impact of stable stratification, such as a boundary layer cooled intensely from below (Monin and Yaglom, 1971). These ideas have been extended to density stratification brought by salt water intrusion, thermohaline stratification, or suspended sediment in marine bottom boundary layers through some revisions. For example, Anwar (1983) and Sanford et al. (1991) applied log-linear profiles to explain the stratification in the Great Ouse and Chesapeake Bay estuaries. Adams and Weatherly (1981) examined the stably stratified oceanic bottom boundary layer using the Mellor-Yamada turbulence closure (MY) model (Mellor and Yamada, 1982) and suggested an expression for the near-bottom velocity profile, which linked the reduction of the Karman coefficient to the constant Richardson's number throughout depth. Friedrichs and Wright (1997) applied the improved log-quadratic velocity profile to obtain realistic estimates of the bottom stresses, which was usually overestimated by the classical log profiles. Herrmann and Madsen (2007) examined the sediment-induced stratification effects on the velocity profiles and sediment concentration distribution in a steady and uniform turbulent flow. However, the above studies neglected the horizontal density gradient effect on velocity profiles.

In numerous estuaries, the salinity distribution changes alternatively between being well-mixed and being partially-stratified within tide cycles. In addition, the regime of the estuarine mixing may vary with the different locations of the estuary according to the distance from the river mouths. As a result, the influence of horizontal density gradient and stratification (by vertical density gradient) may coexist and interact in the hydrodynamic mechanics of the estuarine circulation. Unfortunately, most of the former theoretical formulae concentrated on either the horizontal density gradient (e.g., Hansen and Rattray (1965); Burchard and Hetland, (2010)) or stratification (e.g., Friedrichs and Wright (1997); Herrmann and Madsen (2007)). The theoretical expression of the velocity profile explicitly including both factors is rarely presented. Although an increasing number of numerical simulations can take multiple factors into account and obtain agreeable results with the measurements (Mazumder and Ghoshal, 2006), they are incapable of differentiating the mechanism of the horizontal density gradient and the stratification influence as explicitly as the theoretical formula do. Moreover, some boundary layer parameters such as friction velocity, roughness length and drag coefficient in estuarine environments, estimated based on theoretical velocity profiles, are required for: (i) defining flow conditions at the sediment-water interface; and (ii) predicting sediment and contaminant transport rates (Collions et al., 1998).

In this paper, we investigate more closely into the impact of the horizontal density gradient on unstratified and stably stratified boundary flow through new analytic solutions of shearing stress distribution, viscosity and velocity profiles. Thus, we have restricted our attention to well-mixed and periodically stratified estuaries, neglecting the earth's rotation, wind stress and advection. For the flow at peak flooding or ebbing time, the tidal acceleration is weak and can be ignored. The tidal straining induced by tidal asymmetry is also excluded for the instantaneous flow.

The paper is organized as follows: First, the shearing stress profile is derived, including the interaction between the horizontal density gradient and the bottom friction. Second, the analytic velocity profiles considering the horizontal density gradient and stratification are proposed according to the Prandtl's mixing length and Monin-Obukhow's stratification theory. Third, a series of ideal numerical simulations using vertical parameterization through Mellor-Yamada turbulence closure (MY) model (Mellor and Yamada, 1982) is performed to validate the theoretical results. The fitting results indicate that, although some differences appear between the MY model and the Prandtl's mixing model, the analytic velocity profile fits the numerical results satisfactorily. Moreover, the new-proposed velocity profiles can be applied to determine the valid range of the log-fit with enough accuracy for estimating the friction velocity in baroclinic flows. While the log-fit is not accurate enough, the estimating error can be given.

2. Theory

2.1. The vertical shearing stress

The momentum equation regardless of earth's rotation along the *x*-coordinate is

$$\frac{\partial u}{\partial t}+\frac{\partial uw}{\partial x}+\frac{1}{\rho_0}\frac{\partial P}{\partial x}-\frac{1}{\rho_0}\frac{\partial \tau}{\partial z}=0 \tag{1}$$

with the velocity u and w along the x-coordinate and z-coordinate, the pressure P, the shear stress τ along the x-coordinate and the averaged water density ρ_0.

According to the static pressure assumption, the pressure is given by

$$\frac{1}{\rho_0}\frac{\partial P}{\partial x}=g\frac{\partial \varsigma}{\partial x}+\frac{g}{\rho_0}\int_z^{\varsigma}\frac{\partial \rho}{\partial x}dz \tag{2}$$

where ς denotes the water elevation,g denotes the gravity acceleration and ρ is the water density at a certain depth.

Substituting (2) into (1) and neglecting the non-linear advection term and local acceleration term, one can obtain

$$\frac{1}{\rho_0}\frac{\partial \tau}{\partial z}=g\frac{\partial \varsigma}{\partial x}+\frac{g}{\rho_0}\int_z^{\varsigma}\frac{\partial \rho}{\partial x}dz \tag{3}$$

By integrating Eq. (3) in conjunction with the boundary condition and eliminating the barotropic pressure by Eq. (1), the solution of the shear stress is obtained as

$$\tau=\tau_b(1-\sigma)-0.5gD^2\frac{\partial \rho}{\partial x}\left(\sigma^2-\sigma\right) \tag{4a}$$

or

$$\frac{\tau}{\tau_b}=(1-\sigma)-Ri_x\left(\sigma^2-\sigma\right) \tag{4b}$$

with the bottom shear stress τ_b, the dimensionless vertical height $\sigma=\frac{h+z}{D}$, the static water depth h and the instantaneous water depth D.

The modified horizontal Richardson number is defined as