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# Latent resonance in tidal rivers, with applications to River Elbe

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ABSTRACT

We describe a systematic investigation of resonance in tidal rivers, and of river oscillations influenced by resonance. That is, we explore the grey-zone between absent and fully developed resonance. Data from this study are the results of a one-dimensional numerical channel model applied to a four-dimensional parameter space comprising geometry, i.e. length and depths of rivers, and varying dissipation and forcing. Similarity of real rivers and channels from parameter space is obtained with the help of a 'run-time depth'. We present a model-channel, which reproduces tidal oscillations of River Elbe in Hamburg, Germany with accuracy of a few centimetres. The parameter space contains resonant regions and regions with 'latent resonance'. The latter defines tidal oscillations that are elevated yet not in full but juvenile resonance. Dissipation reduces amplitudes of resonance while creating latent resonance. That is, energy of resonance radiates into areas in parameter space where periods of Eigen-oscillations are well separated from the period of the forcing tide. Increased forcing enhances the re-distribution of resonance in parameter space. The River Elbe is diagnosed as being in a state of anthropogenic latent resonance as a consequence of ongoing deepening by dredging. Deepening the river, in conjunction with the expected sea level rise, will inevitably cause increasing tidal ranges. As a rule of thumb, we found that 1 m deepening would cause 0.5 m increase in tidal range.



## 1. Introduction

There are a number of coastal regions on earth the Eigen-oscillations of which being in resonance with the tide. This applies to, for example, the Bay of Fundy in Canada, the Bristol Channel with River Severn in the UK, and the Bay of St Michel in France (Garrett, 1972; Godin, 1993; Liang et al., 2014). Apart from cases with confirmed full and spectacular resonance, there may be other regions with only weak or latent resonance which, however, remain largely understudied. The term 'latent resonance' should be understood in the sense of not fully developed yet potential or juvenile resonance. Resonance in a tidal river or a bay is expected to occur if the quarter wave length criterion applies (Defant, 1961; Godin, 1993; Proudman, 1953). That is, if the length of the oscillator equals one quarter of the wave length of the agitating tide. Amplitudes of a resonant oscillator have the character of standing waves. In case a wave has a nodal point at the mouth of the oscillator, full resonance will occur at its end.

Amplitudes at the end of an oscillator exceeding those of its forcing may indicate beginning resonance. Since the 1970s, tidal ranges in the port of Hamburg at River Elbe in the north of Germany rose by about a meter. Today, the tidal range at Hamburg, some 130 km inland, is 70 cm higher than at the river mouth whereas thirty years ago it was the opposite. The amplification factor (elevation ratio river end versus mouth) is 1.23 with tidal ranges still growing. This is a manifestation of anthropogenic change caused by dredging. In deepened topography dissipation, progression of the tidal wave and thus Eigen-oscillations will change. Will ongoing dredging gradually transform the river into a state of resonance? If what is observed in the river is resonance, is it still *in statu nascendi*, i.e. virginal, or is it already advanced?

Quite a few coastal regions and rivers may, in the future, reach a state of latent resonance as a result of predicted sea level rise due to climate change. To help diagnose resonance in tidal rivers under modified environmental conditions, we here present a systematic study that allows locating resonance in a great variety of tidal rivers. We particularly look at tidal oscillations that are influenced by not yet fully developed resonance. That is, oscillations in latent resonance. Hence, we are interested in investigating the grey-zone around resonance.

## 2. Materials and methods

It would be desirable to have an instrument which allows an estimation of the resonant state of any river. Thus, rather than focussing our investigation on possible resonance in River Elbe, we preferred to carry out a systematic study encompassing the most important parameters

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affecting resonance in tidal rivers. Parameters are geometry of a river, i.e. length and depth, dissipation, and amplitude of forcing. Tides in a river are long-wave phenomena with river lengths often shorter than the tidal wave length. We consider rivers as oscillators and are only interested in oscillations of long waves. Therefore, deliberately neglecting any details in the complexity of river topographies like deltas, navigational channels, islands, and side arms, we approximate rivers as one-dimensional channels with uniform depth and given length. No river on earth has a uniform depth but we obtain similarity between real rivers and model channels by utilising a 'run-time depth', which is derived by combining the observed phase speed of a tidal wave in a river with the phase speed of shallow water waves. We will use this similarity parameter below.

Parameters river length ($L$), depth ($H$), dissipation ($R$), and amplitude ($A$) of tidal forcing form a four-dimensional space. River lengths $L$ range between 50 and 200 km with increments of 5 km. Depth $H$ increases every meter from 5 m to 30 m. For the non-dimensional friction parameter $R$, the following values, in decaying order, apply: $2.5\cdot10^{-3}$, $1.75\cdot10^{-3}$, $1.0\cdot10^{-3}$, $5.0\cdot10^{-4}$, and $1.0\cdot10^{-4}$. The first $R$ value has extensively been used in 2D vertically integrated storm surge models. We consider it as an upper limit because the spatial increment of our model is two orders of magnitude smaller. Amplitudes $A$ of tidal forcing increase from 0.5 m to 2 m with increments of 0.5 m.

Each quadruple ($L$, $H$, $R$, $A$) in parameter space represents a 'river' with given forcing and dissipation. Tidal oscillations for the 16,120 elements in parameter space are provided by a one-dimensional vertically integrated non-linear numerical channel model, our oscillator. For each case, i.e. quadruple the model, while starting from rest, is run over 8 tidal periods to reach cyclic stationary state. Tidal ranges at river end from the respective last tidal period are stored. With few exceptions, these tidal ranges are the only results produced by the model that will be used in this study. That is, we mostly confine ourselves to oscillations at the river end that are modulated by forcing, geometry, and dissipation. Only to check compliance with the quarter wave length criterion tidal ranges along channels will be considered. The model was applied with both semi-diurnal and diurnal forcing.

The governing equations of the channel model notated in momentum form are

Equation of motion

$$\partial U/\partial t + gH\partial\zeta/\partial x = -R\ \ U|U|/H^2$$

Equation of continuity

$$\partial\zeta/\partial t = -\partial U/\partial x$$

With $U = uH$ momentum ($m^2/s$); $u$ velocity (m/s); $\zeta$ surface elevation (m); $H = D + \zeta$ actual water depth (m), $D$ undisturbed depth (m); $R$ seabed friction parameter (./.); $g$ earth's gravity acceleration ($m/s^2$).

Non-linearity in the model arises from seabed friction, pressure gradient, and divergence in continuity equation. In all of these terms, the actual water depth $H = D + \zeta$ appears. Apart from quadratic seabed friction, this is the main source for non-linearity in the model bearing in mind that under resonance, undisturbed depth $D$ and surface elevation $\zeta$ may be in the same order of magnitude. The channel model is approximated on a staggered Arakawa C-grid (Arakawa and Lamb, 1977) with a space/time resolution of $\Delta x = 250$ m and $\Delta t = 30$ s, respectively. Implicit schemes approximate both surface oscillations and seabed friction (Backhaus, 1983). The implicit scheme for the free surface is neutral with regard to damping since future and presence time levels of pressure gradients and momentum divergence are equally weighted (Backhaus, 1983). This has been checked against explicit schemes.

For the approximation of seabed friction in the equation of motion, we use a semi-implicit approach in which momentum $U$ at future ($n + 1$) and presence time level ($n$) coexist (underlined):

$$U^{n+1} = U^n - \Delta t\ \ gH(\zeta_E - \zeta_W)/\Delta x - \Delta t\ \ R\ \ \underline{U^{n+1}|U^n|}/H^2,$$

with $E$ and $W$ compass-rose indexing, and $\Delta t$ and $\Delta x$ time and space increments.

This results in:

$$U^{n+1} = (U^n - \Delta t\ \ gH(\zeta_E - \zeta_W)/\Delta x)/\left(1 + \Delta t\ \ R|U^n|/H^2\right)$$

On the other hand, were momentum defined only at time level $n$, i.e. $\Delta t\ R\ U^n\ |U^n|/H^2$ the approximation of friction in the equation of motion leads to:

$$U^{n+1} = U^n\left(1 - \Delta t\ \ R|U^n|/H^2\right) - \Delta t\ \ gH(\zeta_E - \zeta_W)/\Delta x.$$

This approximation may cause a change of sign in flow direction if, under extreme conditions, the positive definite property $\Delta t\ R|U^n|/H^2$ becomes > 1. This violation of physics would soon let the model fail. It is readily seen that the above semi-implicit version of seabed friction is void of this error because $1 + \Delta t\ R|U^n|/H^2$ is always > 1.

The implicit components of the model make it very robust. This is a helpful and necessary attribute since there are regions with high resonance in parameter space where tidal elevations are in the same order as water depths. It turned out that the lowest value of friction parameter $R$ ($1.0\cdot10^{-4}$) is the smallest feasible. For a lower value, it was impossible to explore the entire parameter range because the model exploded in resonant regions. Fong and Heaps (1978) also used a one-dimensional numerical model to investigate resonance. However, they did not carry out a systematic study but focussed on Bristol Channel.

## 3. Results and discussion

To explore tidal oscillations in the four-dimensional parameter space, it is advisable to isolate sub-domains. We expect a competition between forcing, i.e. tidal amplitude $A$, and dissipation $R$ which can best be studied in the (LH) sub-domain defined by geometry, i.e. length $L$ and depth $H$. Resonance, in $LH$ space, is expected to occur where the tidal period is close to the period of Eigen-oscillations defined by $L$ and $H$. The seiche period $Ts$ for a one-sided open channel is given by the Merian formula: $Ts = 4$ L/C, with C phase speed of a shallow water wave for depth $H$, and $L$ channel length. Thus, for each duple in LH space, we know its respective seiche period. We at first investigated resonance with the M2 tide. Below, we will use the River Elbe as example of how to utilise data from parameter space.

### 3.1. Resonance in LH space

To allow judging the effect of dissipation on resonance in the LH sub-domain, we contrast results for the respective lowest and highest friction parameter $R$ for one and the same forcing. Results for three forcing amplitudes $A$ (0.5, 1.0, and 2.0 m) are presented to also allow observing changes in resonant features caused by increased forcing (Figs. 1–3). The interaction of forcing and dissipation modifies amplitudes and patterns of resonance in LH space, thereby creating latent resonance. For low-dissipation full resonance characterised by tidal ranges in excess of forcing (Fig. 1 a), is found at LH pairs, i.e. Eigen-periods, close to the M2 period (black line in Figs. 1–3). A narrow sable-like ridge of tidal ranges in full resonance runs diagonally through LH space (Fig. 1a). With increased forcing, this ridge both heightens and broadens (Figs. 1–3). Dissipation lowers the height of the ridge but likewise causes lateral spreading towards initially non-resonant regions in LH