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Evaluation of the accuracy of an offline seasonally-varying matrix transport model for simulating ideal age

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A B S T R A C T

Newton–Krylov solvers for ocean tracers have the potential to greatly decrease the computational costs of spinning up deep-ocean tracers, which can take several thousand model years to reach equilibrium with surface processes. One version of the algorithm uses offline tracer transport matrices to simulate an annual cycle of tracer concentrations and applies Newton's method to find concentrations that are periodic in time. Here we present the impact of time-averaging the transport matrices on the equilibrium values of an ideal-age tracer. We compared annually-averaged, monthly-averaged, and 5-day-averaged transport matrices to an online simulation using the ocean component of the Community Earth System Model (CESM) with a nominal horizontal resolution of $1^\circ \times 1^\circ$ and 60 vertical levels. We found that increasing the time resolution of the offline transport model reduced a low age bias from 12% for the annually-averaged transport matrices, to 4% for the monthly-averaged transport matrices, and to less than 2% for the transport matrices constructed from 5-day averages. The largest differences were in areas with strong seasonal changes in the circulation, such as the Northern Indian Ocean. For many applications the relatively small bias obtained using the offline model makes the offline approach attractive because it uses significantly less computer resources and is simpler to set up and run.

1. Introduction

Tracers in the deep ocean take thousands of years to equilibrate with their sources and sinks (e.g. Primeau and Deleersnijder, 2009). As a result, ocean general circulation models that aim to simulate climatological tracers need long spin-up runs before reaching their equilibrium state. Even for models of moderate spatial resolution the computational burden is significant. The problem is further compounded by the CFL stability conditions, which impose ever smaller time-step size as resolution increases. Consequently, the push for models of ever increasing resolution means that the required computational resources will continue to outpace the expected increases in computational resources for the foreseeable future.

To avoid the excessive computational burden associated with the long time to tracer equilibrium, Li and Primeau (2008) and also Khatiwala (2008) showed that a Newton–Krylov algorithm can be used to greatly decrease the computational time needed to spin up biogeochemical tracers to their seasonally-varying equilibrium state. The method solves for the periodic state of the annual cycle without solving for the transient spin-up of the tracers. This is accomplished by solving the coupled system of algebraic equations that propagate the tracer state of the ocean at the beginning of the year forward in time by exactly one year, such that the same tracer state is achieved at the end of the year. The algorithms proposed by Li and Primeau (2008) and also by Khatiwala (2008) used an offline tracer transport matrix with monthly temporal resolution to simulate the time-dependent seasonal cycle; however, the offline simulation is not an essential part of the algorithm. The Newton–Krylov algorithm can also be applied by using the full dynamical model to simulate the seasonal cycle needed to evaluate the transformation of the tracer (Lindsay, 2016). The tracer transport matrix is only indispensable for the construction of an effective preconditioner. In fact, using the online dynamical model to propagate the model tracer state forward for one year has the advantage of eliminating the errors incurred by approximating the continuous time-dependence of the advection-diffusion operator by a discrete set of 12 monthly transport operators. The main advantages of the offline tracer transport matrix are that it uses significantly fewer computational resources and is in general much simpler to set up

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and run. Other useful additional capabilities are the ease of applying filtering operations to the transport matrix operators; the ease of using subsets of the operators (such as advection only); and the ability to restrict simulations to specific regions.

There is a substantial and growing body of work that is based on a transport matrix constructed by averaging the circulation of an OGCM (e.g. Primeau, 2005; Khatiwala et al., 2005; Kwon and Primeau, 2006; Kriest and Oschlies, 2011; Graven et al., 2012; Prieß et al., 2013; Bardin et al., 2014; Letscher et al., 2015), yet the error incurred by approximating the time-dependence of the circulation by step-wise constant tracer-transport matrices has never been rigorously assessed. The goal of this study is to start to fill in this gap by comparing an equilibrium tracer field obtained with an online simulation to one obtained from an offline matrix-transport model in which the seasonal cycle is represented by (i) an annually averaged matrix, (ii) a set of 12 monthly averaged matrices, and (iii) a set of 73 5-day averaged matrices. The tracer we consider is the ideal age (e.g. England, 1995; Deleersnijder et al., 2001; Primeau, 2005), because it nicely illustrates the long timescales of ocean transport and because it is straightforward to implement without any complicated tracer source-sink functions.

The main objective here is to quantify the difference the averaging of advection and diffusion fields that occurs in the offline approach makes on the computed ideal age, compared to using the online model with the full time resolution of the parent dynamic model. Are the differences using the offline approach sufficiently small that the precomputed transport matrices can be used for biogeochemical sensitivity studies, for calibrating sub-gridscale eddy diffusivity parameterization, or for the spin-up of biogeochemical tracers for the purpose of initializing transient simulations? We lay the groundwork by which to answer these questions quantitatively.

2. Methods

2.1. Ideal age

The ideal age is defined as the mean time since a water parcel last had contact with the surface. It can be computed by solving the following initial value problem

$$\begin{cases} \dfrac{\partial a}{\partial t} + \nabla \cdot [\mathbf{u} - \mathbf{K}\nabla] a = 1, \\ \qquad\qquad \widehat{\mathbf{n}} \cdot \mathbf{K}\nabla a = 0, \ \text{on all solid boundaries}, \\ \qquad\qquad\qquad a = 0 \ \text{at the sea surface}, \ a(t_o) = a_o, \end{cases} \tag{1}$$

where a is the age, $\mathbf{u}$ is the fluid velocity, and $\mathbf{K}$ is the eddy diffusion tensor. The bold notation denotes vectors or tensors. “1” on the right hand side of the equation is an age source or rate of aging, i.e. 1 year per year.

We consider the case where the ocean circulation is perfectly periodic with a period of $\Delta T = 1$ year. This condition is satisfied provided the fluid velocity and eddy diffusivity tensor are both periodic with

$$\begin{aligned} \mathbf{u}(t + \Delta T) &= \mathbf{u}(t), \\ \mathbf{K}(t + \Delta T) &= \mathbf{K}(t). \end{aligned} \tag{2}$$

We are interested in the long-time asymptotic solution to Eq. (1) at which point the transients have died out so that the ideal age is also periodic with $a(t + \Delta T) = a(t)$.

2.2. The dynamical ocean model

For this study we simulate the ocean circulation using the dynamical ocean model component (POP2) of the Community Earth System Model (CESM), version 1.2 (Smith et al., 2010). The physical circulation was “spun-up” from rest for 250 years using the Common Ocean-ice Reference Experiments (CORE) climatological normal year forcing (Large and Yeager, 2004; Griffies et al., 2009; Large and Yeager, 2009), in which the same seasonal cycle is repeated every model year. The temperature and salinity were initialized from the January-mean climatology. Following the spin-up of the dynamics, the dynamical ocean model was run for an additional year during which all quantities needed to construct the model’s advection-diffusion transport operators in matrix form with monthly or 5-day time resolution were saved. The monthly-averaged or 5-day-averaged transport matrices provide representations of the seasonal cycle. These transport matrices can also be averaged to produce the annually averaged transport matrix. Details of the construction of the offline transport matrices from the parent dynamical model are given in Bardin et al. (2014).

The length of the dynamical model spin-up was chosen based on the tradeoff between the use of computational resources and wall-clock time, and the model simulation time needed for the transients in the momentum equations to decay. The 250-year duration of the spin-up is inadequate for the transients in $\mathbf{u}$ and $\mathbf{K}$ to completely die out, but we nevertheless enforce the periodicity of the circulation when we solve Eq. (1). By doing so we suppress the effect of any intrinsic inter-annual variability on the ideal age. For the online model this is done by reinitializing the dynamical-model to the same model-year state conditions at the beginning of each year of iteration as described in Lindsay (2016). For the offline model this is done by recycling the same transport matrices as described in Bardin et al. (2014).

Other details of the model configuration are as follows. The model has a dipolar grid with the convergence point normally found at the geographic North Pole displaced into Greenland; the transition from the grid following latitudinal and longitudinal lines starts at the Equator. The nominal $1^\circ \times 1^\circ$ horizontal grid has a higher meridional resolution of about 0.3° approaching the equator. In the vertical dimension there are 60 depth levels with vertical resolution of 10 m for the top 16 layers, increasing to 250 m near the bottom. The thickness of the top-most layer is allowed to vary from its 10 m thickness to include regional and temporal variations in sea surface height. Details of the numerical methods and discretization are given in the POP Reference Manual (Smith et al., 2010).

Transport of tracers in the model is achieved by a combination of explicitly resolved currents and parameterized sub-gridscale transport processes. Tracer transport is divided into an advection term, a “horizontal diffusion” term, and a “vertical diffusion” term. The advection term includes the explicitly resolved velocity as well as a bolus velocity that arises from the Gent McWilliams (GM) parameterization of mesoscale eddies (Gent and McWilliams, 1990). The advection term also incorporates a sill-overflow parameterization for the deep channel overflows in the Denmark Strait and the Faroe Banks Channel; and for continental shelf overflows in the Weddell and Ross Seas (Briegleb et al., 2010; Danabasoglu et al., 2010). A third-order upwind advection scheme is used to compute the advective tendencies of the explicitly-resolved and bolus velocities. The model also includes a purely diffusive parameterization of mixing by mesoscale eddies in the form of a diffusion tensor that is rotated so that the large “horizontal” diffusion acts along isopycnal surfaces rather than across them (Redi, 1982). Thus the vertical diffusion term combines the vertical component of the along-isopycnal diffusion together with the diffusion from the K-profile parameterization (KPP) (Large et al., 1994), which accounts for the mixing induced by convective instabilities, shear instabilities, destabilizing tendencies associated with vertical gradients of salinity and temperature, as well as tidal mixing. The horizontal diffusion term captures the horizontal component of the along isopycnal diffusion.