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Limited fetch revisited: Comparison of wind input terms, in surface wave modeling

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ABSTRACT

Results pertaining to numerical solutions of the Hasselmann kinetic equation (*HE*), for wind driven sea spectra, in the fetch limited geometry, are presented. Five versions of source functions, including the recently introduced *ZRP* model (Zakharov et al., 2012), have been studied, for the exact expression of S_{nl} and high-frequency implicit dissipation, due to wave-breaking. Four of the five experiments were done in the absence of spectral peak dissipation for various S_{in} terms. They demonstrated the dominance of quadruplet wave–wave interaction, in the energy balance, and the formation of self-similar regimes, of unlimited wave energy growth, along the fetch. Between them was the *ZRP* model, which strongly agreed with dozens of field observations performed in the seas and lakes, since 1947. The fifth, the *WAM*3 wind input term experiment, used additional spectral peak dissipation and reproduced the results of a previous, similar, numerical simulation described in Komen et al. (1994), but only supported the field experiments for moderate fetches, demonstrating a total energy saturation at half of that of the Pierson–Moscowits limit. The alternative framework for *HE* numerical simulation is proposed, along with a set of tests, allowing one to select physically-justified source terms.

1. Introduction

The motivation, for the research presented in the current paper, was to continue the project of finding a firm scientific foundation for the study of wind driven seas.

The most important step in this direction was made in 1962, by K. Hasselmann (Hasselmann, 1962; 1963; Nordheim, 1928), who proposed the kinetic equation for wind waves description

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss} \tag{1}$$

similar to equations used in condensed media physics since the 1920s (Landau and Lifshitz, 1980), where $\varepsilon = \varepsilon(\omega_k, \theta, \vec{r}, t)$ is the wave energy spectrum, as a function of wave dispersion $\omega_k = \omega(k)$, angle θ, two-dimensional real space coordinate $\vec{r} = (x, y)$ and time *t*. S_{nl}, S_{in} and S_{diss} are nonlinear, wind input and wave-breaking dissipation terms, respectively. Hereafter, only the deep water case, $\omega = \sqrt{gk}$ is considered, where *g* is the gravity acceleration and $k = |\vec{k}|$ is the absolute value of wavenumber $\vec{k} = (k_x, k_y)$.

Eq. (1) is widely accepted in the oceanographic community (Young, 1999; Komen et al., 1994) and has several names. It is called the Boltzmann equation (Komen et al., 1994) (while this is not exactly correct), the energy balance equation (Young, 1999), and the radiation balance equation. We will call it the Hasselmann equation (hereafter *HE*) as a tribute to Hasselmann's pioneering work. At the least, this is consistent with part of the community (Janssen, 2009).

The right side of Eq. (1) consists of three terms. The S_{nl} term is completely known. It was consistently derived from Euler equations and describes quadruplets of waves satisfying resonant conditions

$$\vec{k} + \vec{k_1} = \vec{k_2} + \vec{k_3}$$
$$\omega_k + \omega_{k_1} = \omega_{k_2} + \omega_{k_3} \tag{2}$$

In the papers (Zakharov, 2010; Zakharov and Badulin, 2011) we introduced the following splitting of the S_{nl}

$$S_{nl}(\omega, \theta) = F(\omega, \theta) - \Gamma(\omega, \theta)\varepsilon(\omega, \theta) \tag{3}$$

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The explicit expressions for F and Γ are presented in the Appendix. The motivation for this splitting is very simple. The term $F(k)$, for any spectral distribution $\varepsilon(\omega, \theta)$, is an *essentially positive function*. We will soon show that this fact is of fundamental importance.

Kinetic equations similar to the Hasselmann equation are routinely used in different areas of theoretical physics. In all cases, the first and central issue is the description of solutions to the stationary equation

$$S_{nl}(\omega, \theta) = 0 \tag{4}$$

Any solution of this equation can be presented in the form

$$\varepsilon(\omega, \theta) = \frac{F(\omega, \theta)}{\Gamma(\omega, \theta)} \tag{5}$$

As far as $\varepsilon(\omega, \theta) > 0$, for all solutions

$$\Gamma(\omega, \theta) > 0 \tag{6}$$

The function Γ also has another physical sense. In the presence of nonlinear wave ensemble, the dispersion law is undergoing the re-normalization

$$\omega(k) \to \omega(k) + \Delta\omega(k) \tag{7}$$

The re-normalization has real and imaginary parts. The imaginary part is

$$Im\Delta(\omega) = \frac{1}{2}\Gamma(\omega, \theta) \tag{8}$$

Everybody knows that Eq. (4) has solutions with thermodynamic equilibrium.

There is Maxwell distribution in the kinetic gas theory, and Plank distribution in quantum statistical dynamics. Physicists believed, for a long time, that the thermodynamic equilibrium spectra are unique solutions of Eq. (4). This is certainly true, if the entropy of a solution is finite. However, Eq. (4) has a broad class of solutions with infinite entropy, governed by fluxes of conservative quantities—energy, momentum and wave action.

These solutions are now called *KZ* (Kolmogorov–Zakharov) solutions and widely used in different areas of physics (see, for instance Lvov and Newell (2000); C. Connaughton (2004); Galtier (2000); 2003)). The general theory of *KZ* solutions is described in the monograph (Zakharov et al., 1992).

A more advanced development is contained in the paper (Zakharov, 2010). The discovery of *KZ* spectra was recognized by the physical community, by awarding a Dirac medal in 2003, for this development.

The first KZ solution was found by Zakharov and Filonenko in 1966 (the English version of Zakharov and Filonenko (1967) was published in 1967). It is the isotropic solution of the stationary Hasselmann Eq. (4) (the details are presented in Section 4):

$$\varepsilon(\omega) = \frac{\beta_{KZ}}{\omega^4} = C_K\frac{g^{4/3}P^{1/3}}{\omega^4} \tag{9}$$

Here P is the energy flux to the high frequency region. It was soon established, (Katz and Kontorovich, 1975), that the solution Eq. (9) is only “the tip of the iceberg”. Actually, Eq. (4) has a much bigger class of *KZ* solutions, outlined in the paper (Zakharov, 2010; Katz and Kontorovich, 1975). The most interesting and important solutions, governed by fluxes of energy and momentum, are anisotropic. They are not exactly power-like, seeing their ω—dependence deviates from the ω^{-4} law, but only mildly.

Meanwhile, numerous laboratory and field experiments showed that, in the important band of frequency, right behind the spectral peak (approximately for $1.5\omega_p < 3.5\omega_p$), the observed spectra are very close to the ω^{-4} law. Experimental data obtained before 1985 was summarized in the well known paper of Phillips (Phillips, 1985). Since then, a lot of new data has accumulated (see, for instance, Romero and Melville (2010), Resio and Perrie (1989); Resio et al. (2004); Long and Resio (2007), Gagnaire-Renou et al. (2011); Zakharov et al. (2014)). Some other experimental results were cited in the article (Zakharov, 2015a).

Recall that the exact S_{nl} can be derived, rigorously, from the Euler equation.

Opposite to it, the “source function” S_{in}—the energy income from the wind, and the energy dissipation function S_{diss}, due to wave-breaking, are only known approximately. In the oceanographic community, there is no consensus regarding their form. We discuss these questions in Sections 2 and 3 of this paper.

The ambiguity of their proper definitions presents the first major issue for wind wave theory, and hinders development of accurate operational models, as well.

The other important issue is connected with S_{nl} collision term numerical simulation. It is the complex, non-linear, operator, with deep internal symmetries. Several S_{nl} simulation algorithms are available, at the moment, for example: Webb–Resio–Tracy (*WRT*) (Tracy and Resio, 1982; Webb, 1978) (also, see important paper (Resio and Perrie, 1991)), Lavrenov (Lavrenov, 2010) and Masuda (Komatsu and Masuda, 1996). The Van-Vledder version of the *WRT* algorithm (Hwang, 2007) has already been included in the *WavewatchIII* and *SWAN* models, for more than a decade.

All of the above algorithms provide reliable results, but are too slow to provide simultaneous *HE* solutions of the Eq. (1) for tens of thousands of spatial points, faster than real time, as is required by operational wave forecasting. Because of this, existing operational models use much faster substitutes for S_{nl}, in the form of *DIA* and its analogs. This is not fatal, as long as the number of quadruplet configurations used in *DIA* is large enough. However, what is wrong is the commonly practiced “tuning” of the *DIA* algorithm parameters, in the operational models.

We must stress, however, that we do not discuss the good and bad sides of different modifications of *DIA* models. The only results discussed are those obtained from the numerical algorithm for solving the exact Hasselmann equation. This code is a modification of the WRT algorithm . We hereby call it *XNL*.

We insist that a correct definition of the source function is necessary, and we assert that it is possible to perform these corrections, without new theoretical constructions or new difficult experiments. It is sufficient to use existing experimental data, in a proper way. For 68 years, starting from a well-known work of Sverdrup and Munk (Sverdrup and Munk, 1947), oceanographers have accumulated a plethora of experimental facts regarding wave growth rate, with respect to winds. Some of those facts were obtained in water tanks, but the most interesting facts come from ocean measurements.

Nowadays, the results of numerous measurements for “fetch limited” field set-ups, where the off shore wind and the waves are quasi-stationary, have been systematized and published (Badulin et al., 2007).

All of those situations are described by the stationary *HE*

$$\frac{\partial\omega}{\partial k}\frac{\partial\varepsilon}{\partial x} = S_{nl} + S_{in} + S_{diss} \tag{10}$$

This equation is solved, in the presented research, for different source functions S_{in} and S_{diss}. Five experiments were carried out, for different wind input functions, and their results were compared to known ocean field experimental data. This comparison actively used the fact that the results of those experiments are well described by *Weak Turbulence Theory* (*WTT*). This theory is explained, in detail, in the monograph (Zakharov et al., 1992), and applications of this theory, to ocean experiments, are presented in the publications (Pushkarev et al., 2003; Zakharov, 2005; Badulin et al., 2005; 2007; Gagnaire-Renou et al., 2011; Zakharov et al., 2014).

The possibility of *WTT* application is based on the fact that, in Eq. (1), S_{nl} is the dominant term. This fact can be explained in the