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On the observability of bottom topography from measurements of tidal sea surface height

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ABSTRACT

The question of whether features of the ocean bottom topography can be identified from measurements of water level is investigated using a simplified one-dimensional barotropic model. Because of the nonlinear dependence of the sea surface height on the water depth, a linearized analysis is performed concerning the identification of a Gaussian bump within two specific depth profiles, (1) a constant depth domain, and, (2) a constant depth domain adjoining a near-resonant continental shelf. Observability is quantified by examining the estimation error in a series of identical-twin experiments varying data density, tide wavelength, assumed (versus actual) topographic correlation scale, and friction. For measurements of sea surface height that resolve the scale of the topographic perturbation, the fractional error in the bottom topography is approximately a factor of 10 larger than the fractional error of the sea surface height. Domain-scale and shelf-scale resonances may lead to inaccurate topography estimates due to a reduction in the effective number of degrees of freedom in the dynamics, and the amplification of nonlinearity. A realizability condition for the variance of the topography error in the limit of zero bottom depth is proposed which is interpreted as a bound on the fractional error of the topography. Appropriately designed spatial covariance models partly ameliorate the negative impact of shelf-scale near-resonance, and highlight the importance of spatial covariance modeling for bottom topography estimation.

1. Introduction

Ocean bottom topography, i.e., the field of ocean depth relative to the undisturbed water surface, is a necessary component for the development of realistic ocean models. Topography influences ocean circulation at a wide range of spatial and temporal scales via kinematics, potential vorticity conservation, and through boundary layer processes. Gridded maps of ocean bottom topography are readily available to ocean modelers; however, their accuracy is poorly quantified (Marks and Smith, 2006) and the impact of topographic error on ocean forecasts is significant (Heemink et al., 2002; Blumberg and Georgas, 2008).

It is within this context that the problem of estimating bottom topography using data assimilation is studied here. The goal is to combine measurements of water surface elevation with hydrodynamic constraints in order to improve topographic maps, particularly on continental shelves where errors in gravimetrically-derived topography are large (Marks and Smith, 2012). The rationale for such an approach is provided by the observation that harmonic constants of the main diurnal (K_1) and semidiurnal (M_2) tides are known from satellite altimetry with 1cm precision, or better, over much of the ocean (Ray and Byrne, 2010; Stammer et al., 2014), which generally corresponds to a fractional error of 1–5%. The idea is that these data could be assimilated into an ocean tide model based on the Laplace Tidal Equations in which the bottom topography is treated as a distributed control parameter, and more accurate estimates of bottom topography could be obtained, particularly in regions where the relative uncertainty in the depth is greater than the relative uncertainty in the satellite-derived tides. This generic approach has been tried previously (Mourre et al., 2004), but generalizing and validating the approach more widely has proved challenging.

The present approach studies the bottom topography estimation problem in a maximally-simplified setting in order to understand the interplay between the dynamics, domain geometry, and data density. An idealized one-dimensional model consisting of shallow water flow over variable topography is used to examine these factors by using the same estimation technique concurrently implemented with more realistic models. Thus, the present paper examines the accuracy with which isolated perturbations to sea-floor

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topography can be identified from measurements of water level alone. The tidal dynamics are approximated by a one-dimensional linear shallow water model. The novelty of studying this simple system is that it allows the nonlinearity connected with the undisturbed water depth to be exhibited, and it permits a more systematic exploration of parameter space than would be otherwise possible.

This paper is organized as follows. The following section describes how variational data assimilation may be applied to identify bottom topography in a one-dimensional wave equation. Following that, the observability of bottom topography is analyzed in two particular cases, (1) a topographic perturbation to a constant depth ocean, and (2) a topographic perturbation to a constant depth ocean adjacent to a near-resonant continental shelf. In Section 3 the observability is defined and quantified by a simple norm, and the observability of the topography is contrasted with the observability of the sea surface height for the constant depth case. This is followed, in Section 4, by an analysis of a near-resonant continental shelf. For both geometries the observability is described as a function of non-dimensional parameters of relevance to applications, namely, the ratio of the spatial data density to the topographic length scale, the ratio of the wavelength of the tide to the topographic length scale, and the ratio of the assumed correlation scale of the topography to the actual correlation scale.

2. A simple model for bottom topography estimation using variational data assimilation

Consider a model for tidal waves within a domain between $x = 0$, the "coastline," where the depth-integrated water transport, U, vanishes; and $x = L$, the "open ocean," where water elevation, η, is specified. Both U and η are taken as complex-valued functions of x, the complex harmonic constants at a given tidal frequency, ω, here equal to $2\pi/12.42\,\mathrm{h}^{-1}$, the main semi-diurnal tidal frequency. The hydrodynamics consist of the continuity and momentum equations,

$$-j\omega U + gH\eta_x + C_d u_f U/H = 0 \tag{1}$$

$$-j\omega\eta + U_x = 0 \tag{2}$$

$$H = H_0(x) + h(x), \tag{3}$$

where $j = \sqrt{-1}$, H is water depth, g is gravitational acceleration, C_d is the bottom drag coefficient, and u_f is a bottom friction velocity which may depend on x. The equations are supplemented by $H = H_0(x) + h(x)$ to emphasize that the bottom topography shall be taken as a control variable, with H_0 its first guess, and h a correction to be determined by data assimilation. The system represents a simplification of the full shallow water system in which bottom stress is linearized, water density is assumed constant, the advective nonlinearity is neglected, and quadratic nonlinearity involving η has been neglected. The specification of the equations is completed by the boundary conditions, $U(0) = 0$ and $\eta(L) = \eta_0$. In this one-dimensional setting the Coriolis term modifies the dispersion relation in a non-essential manner and so rotation is neglected.

The topographic estimation problem is posed in the language of variational state estimation, where the model state consists of (H, U, η). An estimate for the state is sought which is consistent with the dynamics specified above, where adjustments to the bottom topography, h, bring the modeled and observed values of η into agreement, allowing for measurement error. It is assumed that the expected value of h is zero and its spatial covariance is given by C_{HH}. For testing purposes, the true solution $(\widetilde{H}, \widetilde{U}, \widetilde{\eta})$ is known, and measurements of $\widetilde{\eta}$ are given, $d_i = \widetilde{\eta}(x_i) + \epsilon_i$, for $i = 1, \ldots, M$, together the variance of ϵ_i, σ^2, the measurement noise. The covariance C_{HH} shall be represented in terms of a variance, $\sigma_H^2(x)$, and a spatial correlation function, $c_{HH}(x, y)$, as

$$C_{HH}(x, y) = \sigma_H(x) c_{HH}(x, y) \sigma_H(y). \tag{4}$$

Particular models for the variance and correlation shall be discussed below.

The estimator for (H, U, η) is given by the minimizer of the objective function,

$$J(H, U, \eta) = \int_0^L \int_0^L h(x) C_{HH}^{-1}(x, y) h(y)\, dy\, dx + \sum_{i=1}^{M} |\epsilon_i|^2 \sigma^{-2}, \tag{5}$$

where the data error is given by $\epsilon_i = \eta(x_i) - d_i$, and $|\epsilon_i|^2 = \epsilon\epsilon^*$ is defined using the complex-conjugate of ϵ, indicated with the super-script *. Taking the variation with respect to (H, U, η) leads to the following system for the minimizer of J,

$$j\omega\mu + C_d u_f \mu/H - \zeta_x = 0 \tag{6}$$

$$j\omega\zeta - g(H\mu)_x = -\sum_{i=1}^{M} \delta(x - x_i)(\eta(x_i) - d_i)\sigma^{-2} \tag{7}$$

$$\lambda = -g\mu\eta_x^* + C_d u_f \mu U^*/H^2, \tag{8}$$

with boundary conditions $\mu(0) = 0$ and $\zeta(L) = 0$. The auxiliary variables $\mu(x)$ and $\zeta(x)$ are Lagrange multipliers associated with the equalities (1) and (2). The optimal estimate of topography, $H(x) = H_0(x) + h(x)$, is computed from H_0, $\lambda(x)$, the covariance function $C_{HH}(x, y)$, and $h(x)$ using the definition,

$$h(x) = \int_0^L C_{HH}(x, y) Re[\lambda(y)]\, dy, \tag{9}$$

where $Re[\,\cdot\,]$ denotes the real part of its argument.

The objective function is quadratic in h and ϵ_i, but non-quadratic in the variables, H, η and U. Nonlinearity is an important issue, but it will not be emphasized compared to the basic linear structure of the estimation problem. Instead, assume the solution consists of a small perturbation (H', U', η') to a basic state, $(\overline{H}, \overline{U}, \overline{\eta})$. Then the solution of equations (1)–(8) approximately satisfies,

$$-j\omega U' + gH'\overline{\eta}_x + g\overline{H}\eta'_x + C_d u_f U'/\overline{H} - C_d u_f \overline{U}/\overline{H}^2 H' = 0 \tag{10}$$

$$-j\omega\eta' + U'_x = 0 \tag{11}$$

$$H' = (H_0 - \overline{H}) + h'. \tag{12}$$

The topographic correction, $h' = \int_0^L C_{HH} Re[\lambda]$, is once again obtained from the first-order optimality condition for an extremum of $J(H, U, \eta)$ written in terms of the adjoint variables (λ, μ, ζ),

$$j\omega\mu + C_d u_f \mu/\overline{H} - \zeta_x = 0 \tag{13}$$

$$j\omega\zeta - g(\overline{H}\mu)_x = -\sum_{i=1}^{M} \delta(x - x_i)(\eta(x_i) - \eta_i)\sigma^{-2} \tag{14}$$

$$\lambda = -g\mu\overline{\eta}_x^* + C_d u_f \mu \overline{U}^*/\overline{H}^2, \tag{15}$$

with boundary conditions $\mu(0) = 0$ and $\zeta(L) = 0$. If the set, $(\overline{H}, \overline{U}, \overline{\eta})$, used for the linearization solves equations (1)–(3), then the expression for λ may be written as,

$$\lambda = -\mu^* j\omega\overline{U}/\overline{H}\big(1 + 2jC_d u_f/(\omega\overline{H})\big), \tag{16}$$

where the dependence of λ on the basic state fields $\overline{U}$ and $\overline{H}$ is exhibited.