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A two-dimensional depth-integrated non-hydrostatic numerical model for nearshore wave propagation

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ABSTRACT

In this study, we develop a shallow-water depth-integrated non-hydrostatic numerical model (SNH model) using a hybrid finite-volume and finite-difference method. Numerical discretization is performed using the non-incremental pressure-correction method on a collocated grid. We demonstrate that an extension can easily be made from an existing finite-volume method and collocated-grid based hydrostatic shallow-water equations (SWE) model to a non-hydrostatic model. A series of benchmark tests are used to validate the proposed numerical model. Our results demonstrate that the proposed model is robust and well-balanced, and it captures the wet–dry fronts accurately. A comparison between the SNH and SWE models indicates the importance of considering the wave dispersion effect in simulations when the wave amplitude to water depth ratio is large.

1. Introduction

Due to bed topography changes, currents, and interactions with coastal structures, waves that propagate from offshore regions to nearshore regions are subject to shoaling, refraction, diffraction, and breaking. The accurate prediction of these phenomena is of great importance for the study of nearshore hydrodynamics and is crucial for predicting sediment transport and bed deformation processes. Various approaches have been proposed for modeling these phenomena, e.g., by solving the Navier–Stokes equations (NSE), shallow-water equations (SWE), Boussinesq-type equations (BTE), and the SWE with non-hydrostatic pressure corrections (SNHE). All of these approaches have advantages and disadvantages.

Advances in computational technology have facilitated the development of direct numerical simulation (DNS) models for solving the NSE, Reynolds-averaged version (RANS) models for solving the Reynolds-averaged NSE, and large-eddy simulation (LES) models for solving the filtered NSE. The flow details can be resolved using a DNS, RANS, or LES model, and the nonlinearity and wave dispersivity are fully maintained, which is useful for fundamental wave-related mechanistic studies (e.g., Lin et al., 1999; Hu et al., 2012). However, these models have disadvantages. First, they are computationally expensive and thus they are usually limited to small-scale simulations (especially with DNS and LES models). Second, these type of models generally require the implementation of complex algorithms to track the free surface. The implementation of a free surface tracking method requires additional computational time (the simulation domain also needs to be extended above the water surface) and, most importantly, there are also challenging issues related to the methods themselves, e.g., accurately conserving fluid mass and calculating spatial gradients near the free surface (Sussman and Puckett, 2000). Moreover, the irregular bed topography is non-trivial to process when developing a NSE based model.

Compared with the 3D (or vertical 2D) DNS, RANS, or LES models, those for solving the depth-integrated equations are attractive due to their lower computational cost and because it is relatively easier to handle complex boundaries and they are simpler to code. The SWE can be solved effectively based on the theory of hyperbolic conservation laws. By using a shock-capturing numerical scheme, a SWE model without any tunable coefficients can predict the wave breaking phenomenon well; for instance, the wave energy dissipated in the wave breaking process and the maximum wave runup height after the wave breaks can be predicted reasonably well (Li and Raichlen, 2002; Tan and Chu, 2010; Hu et al., 2015). One of the disadvantages of the SWE models for applications in the nearshore area is the lack of representation of wave dispersivity. When short waves propagate over a long distance, significant errors can be predicted in the wave

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phase and amplitude (Grilli et al., 2007; Kazolea and Delis, 2013). Unlike the SWE, the BTE can represent the weak dispersivity of waves propagating in shallow or intermediate depth water. Numerous studies have tried to improve the nonlinearity and dispersivity properties of the BTE. For instance, Gobbi and Kirby (1996) derived a fully nonlinear BTE model with high-order accuracy in terms of dispersion based on a specifically designed dependent variable (a weighted averaged velocity potential at two vertical locations). The BTE, especially the improved versions, contain high-order spatial or temporal derivatives (e.g., a fifth-order spatial derivative in the model proposed by Gobbi and Kirby (1996)), which is a challenging issue for numerical implementation. In addition, it is generally inappropriate to apply the BTE models directly to simulate wave breaking processes in the surf zone as a predefined breaking criterion is required, and it is usual to introduce an artificial viscosity term or a surface roller model to numerically dissipate the wave energy during the wave breaking process (Brocchini, 2013; Mccabe et al., 2013).

To simultaneously represent the wave dispersivity satisfactorily and to avoid calculating high-order spatial derivatives, a SNH model for solving the SNHE was recently proposed by Stelling and Zijlema (2003), who decomposed the pressure into hydrostatic and non-hydrostatic parts, and by assuming appropriate vertical distributions for the non-hydrostatic pressure and vertical velocity, the SNHE was deduced by depth-integrating the RANS with suitable boundary conditions. Using the Arakawa C grid and employing a finite-difference scheme, Stelling and Zijlema (2003) obtained an elliptic equation for the non-hydrostatic pressure. The solution of this equation was then used to compute the horizontal velocity components at the new time level and finally, the water surface level was updated via the free surface kinematic boundary condition. Based on the study of Stelling and Zijlema (2003), various SNH models have been developed using either the non-incremental pressure-correction method (NPCM), incremental pressure-correction method (IPCM) (Guermond et al., 2006), or (fully) fractional step method (FSM) (Chorin, 1968). Note that the names of these solution methods may vary and they are elusive in previous studies, we here follow the definitions presented in Guermond et al. (2006), where the NPCM and IPCM are two versions of the pressure-correction method (Goda, 1979; van Kan, 1986). When solving the SNHE, both the NPCM and IPCM split the overall solution into two sub-steps, where the former solves the SWE (without considering the non-hydrostatic pressure effect) in the first sub-step and computes the non-hydrostatic pressure in the second sub-step, whereas the latter includes the non-hydrostatic pressure effect (approximated) to determine the intermediate velocity components in the first sub-step, as well as computing the pressure difference between the last and new time levels in the second sub-step. The FSM is very similar to the NPCM when solving the SNHE; these two can be distinguished that the NPCM applies a correction to the water depth (or surface elevation) after the non-hydrostatic pressure is solved in the second sub-step, whereas the FSM does not implement this procedure. In the following, we refer to the SNH models using the NPCM, IPCM, and FSM as SNH-NPC, SNH-IPC, and SNH-FSM, respectively. Zijlema and Stelling (2005) and Yamazaki et al. (2009) developed SNH-FSM models using finite-difference schemes with the same grid layout technique as that employed by Stelling and Zijlema (2003). Zijlema and Stelling (2005) also developed a SNH-IPC model and their numerical experiments showed that the SNH-IPC model maintained the wave shape well, while significant wave damping was predicted by their SNH-FSM model. Walters (2005) and Wei and Jia (2013) developed similar SNH models using finite-element schemes, where both employed a semi-implicit method to obtain provisional solutions in the first sub-step. The two models developed by Walters (2005) and Wei and Jia (2013) differed in that the former was a SNH-FSM model whereas the latter was a SNH-NPC model. They also differed in their treatment of the convection terms as the former used a Lagrangian advection method to enhance the numerical stability, whereas the latter adopted a bounded upwind scheme to avoid spurious oscillations. Cui et al. (2012) recently established a SNH-NPC model based on the SWE solver developed by Cui et al. (2010). In Cui et al. (2010), the finite-element algorithm for solving the SWE was implemented in the model called TsunAWI (Behrens, 2008) and it was converted into a finite-volume scheme with specifically defined finite volumes. This SWE model generally performs well, but minor non-physical oscillations in the solution can be predicted around discontinuities (Cui et al., 2010). In addition, the well-balanced property of this SWE model was not verified.

Considering that most SNH models were developed based on SWE solvers using the finite-difference or finite-element schemes, while numerous SWE solvers have been developed based on the finite-volume method, and given that the state-of-the-art of the SWE solvers using the finite-volume method with schemes designed based on the hyperbolic conservation laws are robust and they can capture shocks and discontinuities well, in the present study, we aimed to develop a SNH model based on a SWE solver using the finite-volume method. In particular, we considered the design of a numerical scheme with the well-balanced property. We compared the performance of our proposed model with existing finite-difference or finite-element method-based SNH models.

The remainder of this paper is organized as follows. In Section 2, we describe the governing equations and the numerical algorithms for the SNH model. In Section 3, numerical tests are used to verify various properties of the numerical model. Finally, conclusions and discussions are presented in Section 4.

2. Numerical model

2.1. Governing equations

The 2D SNHE can be derived from the following 3D RANS

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1a}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right), \tag{1b}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right), \tag{1c}$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial vw}{\partial y}+\frac{\partial ww}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{1}{\rho}\left(\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zy}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right)-g, \tag{1d}$$

where u, v, and w are the velocity components in the horizontal x-, y-, and vertical z-directions, respectively; t denotes time; ρ is the flow density; g is the acceleration due to gravity; and $\tau_{x_l x_r}(l,r=1,2,3)$ denote the shear stresses, where x_1, x_2, and x_3 denote the x-, y-, and z-coordinates, respectively. P denotes the pressure, which is decomposed into hydrostatic and non-hydrostatic components as

$$P=p+\Gamma, \tag{2}$$

where p is the hydrostatic pressure defined as $\frac{\partial p}{\partial z}=-\rho g$ and Γ is the non-hydrostatic pressure.

Following Stelling and Zijlema (2003), we assume that Γ and w vary linearly with water depth; hence, their depth-averaged values are $\overline{\Gamma}=\frac{\Gamma_s+\Gamma_b}{2}$ and $\overline{w}=\frac{w_s+w_b}{2}$. In the following, the $\overline{(\cdot)}$ denotes the depth-averaging operator and the subscripts "s" and "b" denote the