Ocean Modelling 96 (2015) 221–225

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Random wave-induced current on mild slopes

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ARTICLE INFO

Article history:
Received 14 April 2015
Revised 7 August 2015
Accepted 22 October 2015
Available online 30 October 2015

Keywords:
Surface gravity waves
Stokes drift
Sloping beds
Random waves
Stochastic approach

ABSTRACT

This paper provides a simple analytical method for calculating the wave-induced current due to long-crested random waves on mild slopes. The approach is based on assuming the waves to be a stationary random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes. An example is included to demonstrate the application of the analytical method for practical purposes using data typical for field conditions; the significant values of the surface Stokes drift and the volume Stokes transport are calculated. The present results can be used to make assessment of the random wave-induced current based on available wave statistics.

1. Introduction

The Stokes drift represents an important transport component of ocean circulation models. This drift current, which has a steep gradient near the surface, is to a large extent responsible for the transport of natural habitants such as plankton and larvae, as well as pollutants such as contaminated ballast water from ships and oil spills. It also plays an important role in mixing processes across the interface between the atmosphere and the ocean. The Stokes drift is obtained as the mean Lagrangian velocity yielding the water particle drift in the wave propagation direction; it has its maximum at the surface, decreasing with the depth below the surface. The total mean mass transport is obtained by integrating the Stokes drift over the water depth, also referred to as the Stokes transport by Rascle et al. (2008). More details of the Stokes drift are given by e.g. Dean and Dalrymple (1984).

The Stokes drift and the Stokes transport were originally defined for regular waves. However, among the early works on wave-induced drift currents are those of Chang (1969), Kenyon (1969) and Wu (1975) where the first two references considered the Stokes drift for random waves. Chang (1969) performed theoretical and laboratory experiments of the mass transport for long-crested random waves in deep water, while Kenyon (1969) extended the theoretical part of Chang (1969) by giving the Stokes drift for short-crested random waves for an arbitrary constant water depth. Wu (1975) performed systematic laboratory measurements of wind- and wave-induced drift currents for deep water waves. Furthermore, the characteristic quantities for the Stokes drift and the Stokes transport for random waves in terms of the sea state parameters, such as significant wave height and characteristic wave period, are also defined (see e.g. Rascle et al., 2008; Webb and Fox-Kemper, 2011). A global database for parameters associated with ocean surface mixing and drift including the surface Stokes drift and the Stokes transport among other parameters by performing wave hindcast of the parameters was described by Rascle et al. (2008). The hindcast results of Rascle et al. (2008) were improved by Rascle and Ardhuin (2013) using new parameterizations of the physical processes involved (see their 2013 paper and the references therein for more details). Relationships between the wave spectral moments and the Stokes drift in deep water at an arbitrary elevation in the water column were considered by Webb and Fox-Kemper (2011), and inter-comparisons were made using different spectral formulations. Myrhaug (2013, 2015) presented bivariate distributions of significant wave height with surface Stokes drift and Stokes transport. Myrhaug (2013) also presented bivariate distributions of spectral peak period with these two Stokes drift parameters. Based on this some statistical aspects of the Stokes drift parameters together with example of results corresponding to typical field conditions were presented. Myrhaug et al. (2014) presented bivariate distributions of wave height with surface Stokes drift and Stokes transport for individual long-crested random waves in deep water including finite bandwidth effects. Based on this, statistical aspect of these Stokes drift parameters were presented and discussed. Myrhaug and Holmedal (2014) provided an analytical tool which can be used to calculate characteristic statistical values of

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http://dx.doi.org/10.1016/j.ocemod.2015.10.007
1463-5003/

the surface Stokes drift and the Stokes transport beneath individual long-crested and short-crested random waves in deep water and in finite constant water depths.

The purpose of this study is to provide a simple analytical method which can be used to give estimates of statistical values of the wave-induced current due to long-crested random waves on mild slopes. The formulation is valid for finite water depths, but here examples are given based on the shallow water approximation. The statistical values considered are the significant values of the surface Stokes drift and the Stokes transport. This is achieved by assuming the waves to be a stationary random process and adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes. Due to lack of data to compare with, example calculations are included to demonstrate how this analytical method can be used to make assessment of the wave-induced current based on available wave statistics. Thus the present paper extends the authors′ previous works to long-crested random wave-induced current on mild slopes.

2. Background for regular waves

Following Dean and Dalrymple (1984) the mean (time-averaged) Lagrangian mass transport at an elevation z_1 in the column in finite water depth h is given as

$$\bar{u}_L = \frac{ga^2k^2}{\omega}\frac{\cosh 2k(z_1+h)}{\sinh 2kh} \quad (1)$$

Here, g is the acceleration due to gravity, a is the linear wave amplitude, k is the wave number corresponding to the cyclic wave frequency ω given by the dispersion relationship $\omega^2 = gk\tanh kh$. Eq. (1) indicates that the water particles drift in the wave propagation direction; this drift has its maximum at the mean free surface $z_1 = 0$ and decreases towards the bottom as $z_1 \to -h$.

In deep water Eq. (1) reduces to

$$\bar{u}_L = \frac{ga^2k^2}{\omega}e^{2kz_1}; \omega^2 = gk \quad (2)$$

In shallow water Eq. (1) reduces to

$$\bar{u}_L = \frac{a^2}{2h}\sqrt{\frac{g}{h}} \quad (3)$$

where the dispersion relationship $\omega = k\sqrt{gh}$ has been used. In this case it should be noted that $\bar{u}_L$ is independent of z_1 and ω.

The Lagrangian mass transport is often referred to as (surface) Stokes drift.

The total mean (time- and depth-averaged) mass transport is given as (Dean and Dalrymple, 1984)

$$M = \frac{\rho g a^2 k}{2\omega} \quad (4)$$

where ρ is the density of the fluid.

In deep water Eq. (4) is used with $\omega^2 = gk$, and in shallow water Eq. (4) reduces to

$$M = \frac{\rho}{2}\sqrt{\frac{g}{h}}a^2 \quad (5)$$

by using $\omega = k\sqrt{gh}$. In shallow water it is noted from Eqs. (3) and (5) that $M/\rho = h\bar{u}_L$ and independent of ω.

M is often referred to as the (volume) Stokes transport. More details of the Stokes drift and the Stokes transport are given by Dean and Dalrymple (1984).

3. Present analytical calculation of random wave-induced drift on mild slopes

Here a stochastic approach will be outlined following the one given by Myrhaug et al. (2014), except for the modification associated with that for mild slopes by adopting the Battjes and Groenendijk (2000) wave height distribution for long-crested random waves. It is assumed, as a first approximation, that the wave-induced current valid for a horizontal bed given in the previous section can be applied for mild slopes as well. This should represent a reasonable compromise between accuracy and simplicity, i.e. the wave-induced current formulas are assumed to be valid for individual random waves on mild slopes. Overall, this is a similar approach to that used in transforming waves over slowly varying seabed conditions. Thus, the local changes are taken into account by using the Battjes and Groenendijk (2000) distribution. For mild slopes the relative change in water depth should be of the order of 1%, i.e. the slope should be milder than about 1:50.

3.1. Theoretical background

In a sea state with stationary random waves in finite water depth Eqs. (1) and (4) can be taken to represent the Stokes drift and the Stokes transport, respectively, associated with single random waves with wave amplitude a and frequency ω, using that $a = H/2$ and $\omega = 2\pi/T$ where H is the wave height and T is the wave period of the single random waves. Thus, if the joint *pdf* (probability density function) of H and T is known, then the statistical properties of the Stokes drift and the Stokes transport can be obtained, e.g. as given by Myrhaug et al. (2014) for the surface Stokes drift and the Stokes transport in deep water.

In shallow water the Stokes drift and the Stokes transport are given in Eqs. (3) and (5), respectively. Thus, the non-dimensional Stokes drift for individual random waves in shallow water, $u = \bar{u}_L/u_{Lrms}$, is given as

$$u = \hat{H}^2 \quad (6)$$

where

$$u_{Lrms} = \frac{1}{8h}\sqrt{\frac{g}{h}}H_{rms}^2 \quad (7)$$

Here H is made dimensionless by taking $\hat{H} = H/H_{rms}$, where $H_{rms} = 2a_{rms}$ is the *rms* (root-mean-square) value of H, and a_{rms} is the *rms* value of a.

Similarly, the non-dimensional Stokes transport for individual random waves in shallow water, $m = M/M_{rms}$, is given as

$$m = \hat{H}^2 \quad (8)$$

where

$$M_{rms} = \frac{\rho}{8}\sqrt{\frac{g}{h}}H_{rms}^2 \quad (9)$$

Now the Battjes and Groenendijk (2000) parametric wave height distribution based on laboratory experiments on shallow foreshores is adopted. This cumulative distribution function (*cdf*) is composed of two two-parameter Weibull distributions of the non-dimensional wave height $\hat{H} = H/H_{rms}$:

$$P(\hat{H}) = \begin{cases} P_1(\hat{H}) = 1 - \exp\left[-\left(\frac{\hat{H}}{\hat{H}_1}\right)^{k_1}\right]; & \hat{H} < \hat{H}_{tr} \\ P_2(\hat{H}) = 1 - \exp\left[-\left(\frac{\hat{H}}{\hat{H}_2}\right)^{k_2}\right]; & \hat{H} \geq \hat{H}_{tr} \end{cases} \quad (10)$$

where $k_1 = 2$, $k_2 = 3.6$, $\hat{H}_1 = H_1/H_{rms}$, $\hat{H}_2 = H_2/H_{rms}$, $\hat{H}_{tr} = H_{tr}/H_{rms}$. Here H_{tr} is the transitional wave height corresponding to the change of wave height where there is a change of the distribution associated with depth-induced wave breaking, given by

$$H_{tr} = (0.35 + 5.8\tan\alpha)h \quad (11)$$

where α is the slope angle, and H_{rms} is related to the zeroth spectral moment m_0 by

$$H_{rms} = (2.69 + 3.24\sqrt{m_0}/h)\sqrt{m_0} \quad (12)$$