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Multi-layer quasi-geostrophic ocean dynamics in Eddy-resolving regimes

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ABSTRACT

The multi-layer quasi-geostrophic model of the wind-driven ocean gyres is numerically investigated using a combination of long-time runs (200 years) needed for accurate statistics, spatial resolutions (grid interval of less than one kilometer) needed for accurate representation of mesoscale eddies, and large Reynolds number ($Re > 10^4$) needed for more realistic flow regimes. We gradually increased the Reynolds number by lowering the eddy viscosity and analysed the corresponding changes of the large-scale circulation, energetics and eddy fluxes, with the goal to understand how the nonlinear eddy dynamics affects the large-scale ocean circulation, as more and more degrees of freedom become dynamically available. Three- and six-layer configurations of the model are considered in order to understand effects of higher baroclinic modes. A parameter sensitivity study is also carried out to show that the explored flow regime is robust.

As *Re* increases, most properties of the flow show no signs of approaching an asymptote, and the following tendencies are found. The time-mean flow properties tend to an asymptote in the three-layer model but not in the six-layer one, suggesting that higher baroclinic modes are dynamically more active at larger *Re*. The eddy kinetic and potential energies grow faster in the six-layer case. The intensity of the eddy forcing (eddy flux divergence) increases with *Re*. The inter-gyre eddy potential vorticity flux is predominantly northward and up-gradient for all *Re* studied. A comparison of the three- and six-layer model solutions revealed an inhibitory influence of high baroclinic modes on the penetration length of the eastward jet extension of the western boundary currents and on the strength of the adjacent recirculation zones. In large-*Re* regimes, the population of eddies is mostly sustained by the eddy generation at the eastern end of the eastward jet rather than in its central section. Finally, by studying the numerical convergence of the solutions, we found the empirical dependency between the eddy viscosity and the required grid resolution: halving the viscosity requires halving the grid spacing.

1. Introduction

The ocean is one of the largest and least understood components of the global climate system. Being a player of fundamental importance in climate variability, the ocean still anchors the accuracy of climate models. One of the limiting factors is our inability to resolve oceanic submesoscale eddies characterised by the length scale $O(1)$ km. For the time being, there are neither experimental facilities nor mathematical models of the ocean that could provide geoscientists with high-resolution and long-time data coverage permitting to study how different multiscale flow components interact. Whereas detailed global ocean measurements on all scales are out of the question for a long time to come, the leading-edge results in numerical modelling instill confidence that high-resolution eddy simulations may become feasible in the near future. Recently, ocean general circulation models (OGCMs) based on nested grids and operating at high resolutions started to appear (Gula et al., 2015). However, such a possibility is not yet within the reach of the modern OGCMs operating on the planetary scale due to their incapability to operate in fully Eddy-resolving regimes, that is, with resolved scales down to 1 km. Even with the cutting-edge model resolution of 1/12°, the eddies are still only marginally resolved and the simulation times are only tens of years (Marsh et al., 2009; Treguier et al., 2014). Although recent milestone simulations, used the MIT general circulation model, demonstrate the ability of the model to work at very high spatial resolutions (1/146° to 1/50° horizontal grid spacing and 90 vertical levels) and to resolve the eddies (Armstrong et al., 2014), the simulation times are short and systematic exploration of the solution convergence and parameter sensitivity studies are unfeasible (Armstrong et al., 2014). However, yet the lack of immense computational power corners OGCMs into unavoidable eddy parameterisation which, being so far largely inaccurate, remain an Achilles heel of the ocean modelling. This sets a favourable situation for lighter oceanic models to guide the research of ocean eddy dynamics until OGCMs can resolve all important length scales for long-time runs.

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In this work we take this opportunity and consider dynamically viable and featurely enriched quasi-geostrophic (QG) model, which simulates the mesoscale motions well beyond its formal limits of applicability (Mundt et al., 1997; Zurita-Gotor and Vallis, 2009). Our goal and the novelty of this work is to explore the eddy effects for a broad range of Reynolds numbers *Re* so that the flow is increasingly controlled by the explicit nonlinear eddy dynamics rather than by diffusive eddy parameterisation. The other goal is to establish a set of benchmark double-gyre solutions and put forward a methodology for systematic analyses of eddy effects in more advanced, but also much more computationally expensive, primitive-equation ocean models.

There are three precursors to this work. The first one is the work by Holland (1978), who pioneered a two-layer eddy-permitting QG model with the horizontal grid resolution of $dx = 20$ km and showed that dynamically resolved fluctuations feed back on the ocean gyres. The second work is by Barnier et al. (1991) who studied the three- and six-layer double-gyre QG model with the horizontal grid resolution of $dx = 10$ km. The main conclusion of the authors is that the high baroclinic modes play a catalytic role in eddy/mean interactions and, thus, elongate the eastward-jet extensions of the western boundary currents such as the Gulf stream and Kuroshio. The third study is the one by Siegel et al. (2001), in which a benchmark six-layer QG solution of ocean gyres with relatively large *Re* was spun up for six years and run for another three years with the horizontal resolution of $dx = 1.6$ km. The authors concluded that at large *Re* the time-mean kinetic energy is relatively independent of *Re*, but meridional eddy fluxes keep increasing with it. There are also some works studying QG surface dynamics requiring high horizontal and vertical resolutions (e.g. Roullet et al. (2012)), but this dynamics is beyond the scope of our study, where we centre on vertical scales of motion related to the pycnocline and captured by the low baroclinic modes, since these motions are the most important ones for the Gulf stream mesoscale eddy dynamics, which is the main focus of our work.

In our work we continue and extend the past studies by (i) considering much more realistic flow regimes, (ii) reaching much larger *Re*, (iii) refining the horizontal grid resolution down to $dx = 937$ m for more accurate representation of mesoscale eddies, and (iv) by significantly extending the simulation times for much more reliable statistics. Our use of the advanced numerics yields another gaining factor of 4 in terms of the finer spatial resolution (Karabasov et al., 2009), though this factor may be smaller at extremely high resolutions. To summarize, in terms of the dynamically resolved degrees of freedom and achieved simulation years, our benchmark solution is at least 1.5×10^6, 8500, and 500 times more expensive, in terms of the degrees of freedom and simulation time, than the ones in (Holland, 1978), (Barnier et al., 1991), and (Siegel et al., 2001), respectively. We also looked more thoroughly into the time-mean flow and eddy properties and their dependencies on *Re* and studied the dynamic effects of high baroclinic modes.

2. Double-gyre *M*odel

We consider the classical double-gyre QG model, describing idealised midlatitude ocean circulation, in three- and six-layer configurations (denoted as 3L and 6L). The multi-layer QG equations (Pedlosky, 1987; Vallis, 2006) for the potential vorticity (PV) anomaly q in a domain Ω are

$$\partial_t q_i + J(\psi_i, q_i + \beta y) = \delta_{1i} F_{\mathrm{w}} - \delta_{iN}\, \mu \Delta \psi_i + \nu \Delta^2 \psi_i, \qquad i = 1, 2, \ldots, N, \tag{1}$$

where $J(f, g) \equiv f_x g_y - f_y g_x$, and δ_{ij} is the Kronecker symbol; $N = \{3, 6\}$ is the number of stacked isopycnal fluid layers for the 3L and 6L setups with depths (from top to bottom): $H_1 = 0.25$ km, $H_2 = 0.75$ km, $H_3 = 3.0$ km; and $H_1 = H_2 = H_3 = H_4 = 0.25$ km, $H_5 = 1.0$ km, $H_6 = 2.0$ km, respectively. The computational domain Ω is a square, closed, flat-bottom basin of dimensions $L \times L \times 4$ km, with $L = 3840$ km. The asymmetric wind curl forcing (Ekman pumping) drives the double-gyre ocean circulation, and it is given by

$$F_{\mathrm{w}} = \begin{cases} -1.80\, \pi\, \tau_0 \sin(\pi y / y_0), & y \in [0, y_0), \\ 2.22\, \pi\, \tau_0 \sin(\pi (y - y_0)/(L - y_0)), & y \in [y_0, L], \end{cases}$$

with a wind stress $\tau_0 = 0.3\ \mathrm{N\,m^{-2}}$ and the tilted zero forcing line $y_0 = 0.4L + 0.2x,\ x \in [0, L]$. Notice that τ is chosen so that to avoid unrealistically strong eastward jet in low-viscosity (high Reynolds number) regimes. The planetary vorticity gradient is $\beta = 2 \times 10^{-11}\ \mathrm{m^{-1}\,s^{-1}}$, the bottom friction parameter is $\mu = 4 \times 10^{-8}\ \mathrm{s^{-1}}$, and the lateral eddy viscosity ν is a variable parameter specified further below.

The layerwise PV anomaly q_i and the velocity streamfunction ψ_i are dynamically coupled through the system of elliptic equations:

$$q_i = \Delta \psi_i - (1 - \delta_{i1}) S_{i1} (\psi_i - \psi_{i-1}) - (1 - \delta_{iN}) S_{i2} (\psi_i - \psi_{i+1}), \qquad i = 1, 2, \ldots, N, \tag{2}$$

with the stratification parameters S_{i1}, S_{i2} chosen so that the first and the second Rossby deformation radii for the 3L and 6L configurations are $Rd_1 = 40$ km, $Rd_2 = 23$ km; and $Rd_1 = 40$ km, $Rd_2 = 16$ km, $Rd_3 = 11.6$ km, $Rd_4 = 9.8$ km, $Rd_5 = 7.8$ km, respectively. Note that Rd_1 is the same in both configurations as well as H_1. Systems (1 and 2) are augmented with the integral mass conservation constraints (McWilliams, 1977):

$$\partial_t \iint_{\Omega} (\psi_i - \psi_{i+1})\, dydx = 0, \quad i = 1, 2, \ldots, N - 1, \tag{3}$$

with the zero initial condition, and with the partial-slip lateral boundary condition:

$$\partial_{\mathbf{nn}} \psi_i - \alpha^{-1} \partial_{\mathbf{n}} \psi_i = 0, \quad i = 1, 2, \ldots, N, \tag{4}$$

where $\alpha = 120$ km and $\mathbf{n}$ is the normal-to-wall unit vector.

The QG model (1–4) is solved with the high-resolution CABARET method based on a second-order, non-dissipative and low-dispersive, conservative advection scheme (Karabasov et al., 2009). The distinctive feature of this method is its ability to simulate large-*Re* flow regimes at much lower, compared to conventional methods, computational costs. An efficient parallelisation of the QG model allowed us to carry out high-performance computations on uniform horizontal grids of size $G = \{129^2, 257^2, 513^2, 1025^2, 2049^2, 4097^2\}$, where the grid of size $X \times X$ is abbreviated as X^2.

Our horizontal grid resolution is consistent with the vertical one so that the shortest deformation radius is at least marginally resolved with 5–10 grid points. Further simultaneous refinement of the horizontal and vertical resolutions is, of course, desirable, but remains beyond the scape of this paper due to the limit of our computational resources.

Finally, we would like to remind that QG approximation relies on several assumptions, and some of them (smallness of the vertical velocity and density anomalies) including the key one - smallness of the Rossby number - break down for submesoscale motions operating on the scales shorter than the relevant Rossby deformation radius. Given our finest nominal grid resolution of about one km and the fact that numerical schemes typically require 5–10 grid points to represent a length scale with reasonable accuracy (Karabasov et al., 2009), we model the length scales down to 5–10 km, which may be near the edge of formal QG applicability.

3. Analyses of the double-gyre solutions

In this section we describe various properties of the ocean model solutions, study main dependencies of the large-scale flow and mesoscale eddies on the Reynolds number *Re* and define some diagnostics for the next sections. The total basin-average time-mean