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Comparing isopycnal eddy diffusivities in the Southern Ocean with predictions from linear theory

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ABSTRACT

We show that the potential vorticity diffusivity predicted by linear stability analysis (LSA), is the same as a linearized version of Lagrangian cross-stream isopycnal diffusivity. Both can be written in terms of the same expression – the product of the eddy kinetic energy (EKE) and the integral time scale that involves the Lagrangian decay scale γ or the growth rate ω_i of the most unstable wave, and a frequency that is related to the difference of the mean flow speed and real part of the phase speed of the unstable waves.

Diffusivities from LSA are compared to Lagrangian isopycnal eddy diffusivities estimated from more than 700,000 numerical particles in the Southern Ocean of an eddying model. They show different spatial dependency. LSA predicts eddy diffusivities that are enhanced at the steering level where the mean flow speed equals the phase speed of the unstable waves. In contrast, Lagrangian diffusivities exhibit no clear steering level maxima, but are instead surface intensified in many places. The differences between the Lagrangian and diffusivities from LSA can be understood because EKE predicted from LSA differs from the simulated one, and because the estimated decay scale γ is on average about 4 times larger than the largest linear growth rate. The diagnosed Lagrangian integral time scale has maxima at the depth where the mean flow speed equals the phase speed of the most unstable wave, but the diffusivity maxima are shifted towards the surface because the simulated EKE decreases rapidly with depth. Possibilities for a simple parameterization for the diffusivity are discussed.

1. Introduction

Most coarse resolution ocean models parameterize the effects of eddies assuming flux-gradient relationships involving a diffusion tensor, mainly consisting of two parts: the first part seeks to parameterize the advection of properties by an additional eddy driven velocity (sometimes called skew-diffusion), the other part can be interpreted as turbulent mixing of properties along isopycnal surfaces, involving a diffusivity (e.g. Griffies, 1998). At the heart of those parameterizations lies the assumption that a local relationship exists between the unresolved eddy fluxes and the resolved flow. Likely, most mesoscale eddies in the ocean are generated by baroclinic instability of the mean flow. Linear baroclinic instability analysis can be used to predict small amplitude eddy fluxes that would arise if all the energy released from the baroclinically unstable mean flow is locally converted to eddy kinetic energy which is then converted to mixing and dissipation (Green, 1970; Killworth, 1997; Eden, 2011; Vollmer and Eden, 2013). Eddy energy that was generated elsewhere and advected into the region of interest is not taken into account. A large fraction of the eddy field appears as coherent structures with large amplitudes and non-linear processes likely play a large role (Chelton et al., 2011). The effects of the non-local, non-linear nature of eddy-mean flow interaction on energy cascades, eddy properties and eddy fluxes remain unclear (Chen et al., 2014a).

A number of studies have investigated the reproducability of eddy dynamics by linear theory with a focus on propagation speeds (e.g. Chelton and Schlax, 1996; Killworth and Blundell, 2004, 2005; Chelton et al., 2007, 2011). A closure for the eddy fluxes based on linear stability analysis (LSA) is able to correctly reproduce the spatial variations of meso-scale eddy fluxes and diffusivities in simple models (Killworth, 1997; Eden, 2011) and other idealized three-dimensional set-ups (Eden, 2012). However, comparisons of observed or more realistic modeled eddying flows with predictions from linear stability theory show that there is not just one flow regime, linear or highly turbulent, but that the dynamics are highly inhomogeneous (Smith, 2007; Tulloch et al., 2011; Vernaille et al., 2011).

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Here, the focus is on isopycnal diffusivities, while the skew diffusion effect is postponed to a later study. We calculate potential vorticity (PV) diffusivities from linear stability analysis (LSA), which are shown to be identical to Lagrangian isopycnal diffusivities in the linearized Generalized Lagrangian Mean framework of Andrews and McIntyre (1978), and compare them to isopycnal eddy diffusivities derived from Lagrangian particles that were advected by the fully non-linear eddying flow in the Southern Ocean of the realistic 1/10° Parallel Ocean Program (POP). We show that both a linearized version of the Lagrangian diffusivity, discussed by Klocker et al. (2012b) and Klocker and Abernathey (2014), and the potential vorticity diffusivity predicted by LSA, are the same if the Lagrangian decay scale is identified to be equivalent to the growth rate of the unstable waves from LSA. We estimate this decay scale from the Lagrangian trajectories together with the eddy properties from LSA and as simulated by the model.

Linear stability analysis predicts diffusivities that can be enhanced at the steering level, where the phase speed of the unstable baroclinic Rossby waves equals the speed of the mean flow (Green, 1970; Killworth, 1997; Eden, 2011; Vollmer and Eden, 2013). In turn, the theory implies in both cases that eddy diffusivities are suppressed at the surface in strong jets, where mean flow and Rossby wave phase speeds differ significantly. The kinematic interpretation put forward has been that when tracers or floats are quickly advected through the eddies by the mean flow, they do not have time to mix and mixing is suppressed, whereas when the eddies move with the flow, they can effectively mix the tracer. Hence steering levels have been interpreted as regions without mixing suppression, and classical mixing length theory can be extended to include this barrier effect (Ferrari and Nikurashin, 2010; Klocker and Abernathey, 2014).

However, the theory applies to linear Rossby waves, and how the speeds of the non-linear eddies figure in remains under debate. Recently, Klocker and Marshall (2014) argued that observed speeds of the non-linear eddies in the Southern Ocean can be reproduced by linear theory when a Doppler shift by just the depth mean velocity is taken into account. Previous studies in the Southern Ocean using models, idealized configurations and observations have suggested mixing is suppressed at the surface in some ACC jets, but not everywhere (Sallée et al., 2008; Griesel et al., 2010; Naveira Garabato et al., 2011; Griesel et al., 2014). A subsurface maximum sometimes exists, however its significance, extent and magnitude, and whether it is consistent with the depth at which mean flow and phase speeds of unstable linear Rossby waves are the same, remains under debate (Smith and Marshall, 2009; Abernathey et al., 2010; Griesel et al., 2010; Klocker et al., 2012a, 2012b; Griesel et al., 2014; Tulloch et al., 2014; Chen et al., 2014b; LaCasce et al., 2014; Chen et al., 2015).

The Lagrangian diffusivity can be written as the product of the Lagrangian integral time scale, and eddy kinetic energy. At the surface, in the presence of strong jets, the floats are carried through meanders and circle around eddies, leading to oscillations in the velocity autocovariance and large negative lobes, that reduce the Lagrangian integral time scale (Griesel et al., 2010), consistent with the ideas of mixing barriers (Klocker et al., 2012a). Thus, Lagrangian statistics are an ideal tool to investigate the relative importance of eddy kinetic energy and mixing barrier/steering level effects on the diffusivity in a highly non-linear flow, as we will discuss in this paper.

We show here that the depth dependence of the diagnosed Lagrangian diffusivity is different from the one predicted by LSA. We analyze and explain this difference. Diffusivities from linear stability analysis exhibit a clear steering level signature at the depth where the mean flow is equal to the phase speed of maximum growth, since the imaginary part of the phase speed is much smaller than the real part, meaning that the vertical structure of the mean flow determines the structure with depth. On the other hand, the depth dependence of the Lagrangian diffusivity is not dominantly determined by the difference in phase speed and mean flow speed, but also by EKE(z) and the depth dependent Lagrangian decay scale.

Section 2 introduces the background and shows that the PV diffusivity from LSA leads to the same expression as the linearized version of Lagrangian diffusivity if the growth rate of the unstable waves equals the Lagrangian decay scale. Section 3 introduces the model and methodology to calculate the Lagrangian diffusivities from the numerical trajectories, Section 4 discusses the surface distribution of eddy properties and diffusivities, Section 5 explores the depth dependence and Section 6 presents the conclusions.

2. Background

2.1. PV diffusivity from linearized QG

Starting point for the linear stability analysis is the linearized quasi-geostrophic potential vorticity equation around a basic state $\Psi = \overline{\Psi} + \Psi'$, with

$$\partial_t q' + \boldsymbol{U}_h \cdot \nabla q' + \boldsymbol{u}'_h \cdot \nabla Q = A_h \nabla^2 q' \tag{1}$$

$$q' = \nabla^2 \Psi' + \partial_z \left(\frac{f^2}{N^2} \partial_z \Psi' \right) \tag{2}$$

$$= \nabla^2 \Psi' + \Gamma \Psi' \tag{3}$$

with the operator $\Gamma = \partial_z(\frac{f^2}{N^2}\partial_z)$, N and $\boldsymbol{U}_h$ are background stratification and horizontal velocity respectively, q' and $\boldsymbol{u}'_h$ the perturbation PV and velocity, Ψ is the quasigeostrophic streamfunction. Assuming horizontally homogenous conditions the mean PV gradient becomes

$$\nabla Q = \beta \boldsymbol{e}_y - \partial_z \left(\frac{f^2}{N^2} \partial_z \underset{\neg}{\boldsymbol{U}_h} \right) \tag{4}$$

where $\boldsymbol{e}_y$ is the unit vector in the meridional direction. Inserting solutions of the form $\psi' = \Psi_0 \Phi(z) e^{i(k_x + k_y - \omega t)}$, with vertical structure function Φ and constant amplitude Ψ_0 leads to a vertical eigenvalue equation

$$\Gamma \Phi = \left(\frac{\underset{\neg}{\boldsymbol{n}} \cdot \nabla Q}{c - \boldsymbol{n} \cdot \boldsymbol{U}_h + i A_h |\boldsymbol{k}|} + \boldsymbol{k}^2 \right) \Phi, \tag{5}$$

in the interior, and

$$(\boldsymbol{n} \cdot \boldsymbol{U}_h - c) \frac{d\Phi}{dz} = \Phi \frac{d}{dz} (\boldsymbol{n} \cdot \boldsymbol{U}_h) \quad \text{at } z = 0, \tag{6}$$

$$(\boldsymbol{n} \cdot \boldsymbol{U}_h - c) \frac{d\Phi}{dz} = \Phi \left(\frac{d}{dz} (\boldsymbol{n} \cdot \boldsymbol{U}_h) - \boldsymbol{n} \cdot \underset{\neg}{\nabla} \tilde{h} \right) \quad \text{at } z = -h \tag{7}$$

at the boundaries, where $c = \omega / |\boldsymbol{k}|$ is the complex phase velocity, $\boldsymbol{n} = \boldsymbol{k}/|\boldsymbol{k}|$ is the direction of phase propagation, and $\tilde{h} = h f^{-2} N^2|_{z=-h}$. Vectors with the subscript $\neg$ are rotated anticlockwise in the horizontal, e.g. $\underset{\neg}{\nabla} = (-\partial_y, \partial_x)$. The lateral viscosity A_h is related to subgrid-scale friction and is introduced to filter fast growing small scale modes that are often related to dynamically less important surface instabilities. The eigenvalue problem (5)–(7) is solved numerically following Smith (2007) and Vollmer and Eden (2013). Eigenfunctions Φ and eigenvalues ω might be complex. For a positive imaginary part of ω, the amplitude grows exponentially in time. With $\boldsymbol{u}'_h = \underset{\neg}{\nabla} \Psi' = \underset{\neg}{\boldsymbol{k}} \Psi'$ and $q' = \Psi_0(-\boldsymbol{k}^2 \Phi + \Gamma \Phi) e^{i(k_x + K_y - \omega t)}$, the eddy PV flux, averaged over one wave cycle

$$\overline{\boldsymbol{u}'_h q'} = -\frac{\Psi_0^2}{2} \left(\frac{\underset{\neg}{\boldsymbol{n}} \cdot \nabla Q}{c - \boldsymbol{n} \cdot \boldsymbol{U}_h} \right) \Phi^2 c_i \underset{\neg}{\boldsymbol{k}} \tag{8}$$