Ocean Modelling 92 (2015) 1–10

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Ocean wave transmission and reflection by viscoelastic ice covers

Xin Zhao, Hayley H. Shen*

Department of Civil and Environmental Engineering, Clarkson University, Potsdam, NY 13699, USA

ARTICLE INFO

Article history:
Received 20 September 2014
Revised 15 May 2015
Accepted 17 May 2015
Available online 28 May 2015

Keywords:
Ice-covered oceans
Wave propagation
Viscoelastic
Transmission and reflection coefficients
Variational method

ABSTRACT

Modeling ice covers as viscoelastic continua, Zhao and Shen, (2013) applied a two-mode approximate method to determine the transmission and reflection between two different ice covers. This approximate solution considered only two modes of the dispersion relation. In addition, the horizontal boundary conditions were simplified by matching mean values over the interfaces. In this study, we employ a variational method (Fox and Squire, (1990)) to calculate the wave transmission and reflection from two connecting viscoelastic ice covers of different properties. The variational approach minimizes the overall error function at the interface of two ice covers, hence is more rigorous than the previous approximate method that minimized the difference between mean values at the interface. The effect of additional travelling and evanescent modes are also investigated. We compare results from different matching methods, as well as the effects of including additional modes. From this study, we find that additional modes do not always improve the results for our model. For all cases tested, two modes appear to be sufficient. These two modes represent the open-water-like and the elastic-pressure wave-like behavior. The two-mode approximate method and the variational method have similar results except at very short wave periods.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

Due to the rapid decrease of ice cover, wave conditions in the Arctic have intensified (Thomson and Rogers, 2014). In response to increased human activities, especially in the partially ice-covered region, wave models have begun including ice effects. For instance, WAVEWATCH III which used to treat ice covers as islands (Tolman, 2003) now includes three more options. In its latest release (Tolman et al., 2014, pp. 53–62), different physical processes are considered in these options: a constant attenuation rate, an eddy viscosity (Liu and Mollo-Christensen, 1988), and viscoelasticity (Wang and Shen, 2010), all rely on parameterization that awaits further theoretical and observational development. In this study we focus on further developing the viscoelastic theory which envisions an ice cover as a continuum with some elastic property that changes wave speed without damping its energy and viscous property that mainly consumes energy but may also contribute to wave speed change for high frequency components.

Real ice covers are inhomogeneous. Waves propagating between ice covers of different properties will transmit part of their energy and reflect the rest. Based on a thin-plate approach, this phenomenon has been studied extensively between open water and elastic plate and between different elastic plates (Squire, 2007, a review). Assuming that ice covers may be represented as a Voigt linear viscoelastic material, Wang and Shen (2011) studied this transmission/reflection problem between open water and an ice cover using a two-mode approximate method. This method was extended to transmission/reflection between two different ice covered regions in Zhao and Shen (2013). The two-mode approximate method included only two propagating modes closest to the open water waves and ignored all other propagating modes and all evanescent modes permitted by the dispersion relation. Furthermore, matching boundary conditions at the interface of two different regions were only carried out in an average sense.

The two-mode approximate method has the obvious advantage of being simpler and computationally faster than other methods that may include more modes and adopt a more rigorous matching method at the interface. However, its effect on the predicted transmission/reflection is uncertain until we compare the results with a better mathematical procedure that includes more admissible modes and treats the boundary conditions more rigorously. In this study, we examine the effect of including more modes that exist in the dispersion relation, including both propagating and evanescent modes. We also improve the matching criterion by using a variational method as in Fox and Squire (1990). We compare these new results with the two-mode approximate method, and previous studies that assumed ice covers as pure elastic materials.

The organization of this paper is as follows. Section 2 briefly outlines the theoretical formulation of the viscoelastic model. In Section 3, the variational method is presented. Section 4 gives the

* Corresponding author. Tel.: +1 315 268 7985.
E-mail address: hhshen@clarkson.edu (H.H. Shen).

http://dx.doi.org/10.1016/j.ocemod.2015.05.003
1463-5003/© 2015 Elsevier Ltd. All rights reserved.

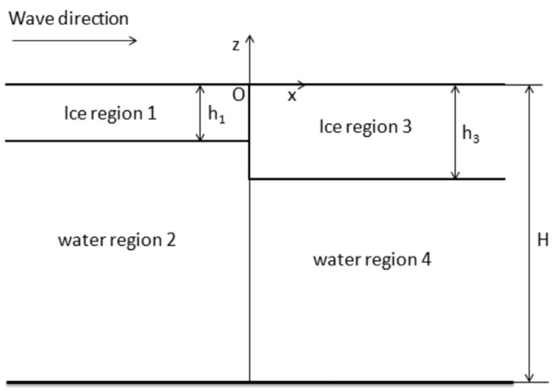

Fig. 1. Schematic of the coordinate frame of the problem.

result of special cases to compare with previous studies for pure elastic ice covers and the analysis on additional modes. Section 5 studies the energy partitions among three main propagating waves in elastic ice and viscous ice. The nature of these modes is also discussed. Section 6 provides details of the error analysis for the current method. The summary and conclusion are given in Sections 7 and 8 respectively. A linear wave regime is assumed in this study.

2. The theoretical formulation

2.1. Definition of the domain

The present study analyzes the same two-dimensional problem as in Zhao and Shen (2013). The sketch for the problem is shown in Fig. 1. The two ice covers are assumed to be fully submerged.

2.2. Governing equations, boundary conditions on horizontal surfaces, and the dispersion relation

In the present study, many more modes from the dispersion relation as derived in Wang and Shen (2010) will be included, hence for clarity, the derivation leading to the dispersion relation is briefly repeated here.

As previously done, for the ice cover we use a Voigt viscoelastic continuum model shown below

$$\tau_{mn} = -p\delta_{mn} + 2GS_{mn} + 2\rho_{ice}\nu\dot{S}_{mn}, \tag{1}$$

where ρ_{ice} is the density of the ice layer; τ_{mn}, S_{mn} and $\dot{S}_{mn}$ represent the stress tensor, the strain tensor and the strain rate tensor, respectively; m and n represent x or z; G and ν are the effective shear modulus and the effective kinematic viscosity of the ice layer, respectively; p is the pressure and δ_{mn} the Kronecker delta. For the regions occupied by an ice cover, either 1 or 3, the equation of motion is

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{\rho_{ice}}\nabla p_i + \nu_{ei}\nabla^2\mathbf{U}_i + \mathbf{g}, \quad i = 1,\ 3, \tag{2}$$

where $\mathbf{U}_i = u_i\hat{e}_x + w_i\hat{e}_z$ is the velocity vector, $\mathbf{g}$ the gravitational acceleration,

$$\nu_{ei} = \nu_i + iG_i/\rho_{ice}\omega, \quad i = 1, 3, \tag{3}$$

is the viscoelastic parameter, and ω is the angular frequency of the incident wave. Using the decomposition with potential function ϕ and stream function ψ for the velocity (Lamb, 1932),

$$\mathbf{U}_i = -\nabla\phi_i + \nabla\times(0,\ \psi_i,\ 0), \quad i = 1,\ 3, \tag{4}$$

we obtain

$$\nabla^2\phi_i = 0, \tag{5}$$

$$\frac{\partial\psi_i}{\partial t} - \nu_{ei}\nabla^2\psi_i = 0, \tag{6}$$

$$\frac{\partial\phi_i}{\partial t} - \frac{p_i}{\rho_{ice}} - \Phi = 0, \quad i = 1,\ 3, \tag{7}$$

Here, $\Phi = gz$ is the gravitational potential.

For the associated water region below the ice covers 1 and 3, i.e. regions 2 or 4, we assume an inviscid fluid. The governing equations are

$$\frac{\partial\mathbf{U}_{i+1}}{\partial t} = -\frac{1}{\rho_{water}}\nabla p_{i+1} + \mathbf{g}, \tag{8}$$

$$\nabla^2\phi_{i+1} = 0, \tag{9}$$

$$\frac{\partial\phi_{i+1}}{\partial t} - \frac{p_{i+1}}{\rho_{water}} - \Phi = 0, \quad i = 1,\ 3. \tag{10}$$

The water velocity is related to the velocity potential only

$$\mathbf{U}_{i+1} = -\nabla\phi_{i+1}, \quad i = 1,\ 3. \tag{11}$$

Next we introduce the boundary conditions at the horizontal interfaces between air–ice, air–water, and water-sea floor. These conditions between regions 1 and 2 are identical to those between 3 and 4.

No stress at the air–ice interface

$$\tau_{xz,i} = \rho_{ice}\nu_{ei}\left(\frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x}\right) = 0,\ \tau_{zz,i} = -p_i + 2\rho_{ice}\nu_{ei}\frac{\partial w_i}{\partial z} = 0,\ z = 0,\ i = 1,\ 3. \tag{12}$$

Stress continuity at the ice–water interface

$$\tau_{zz,i} = -p_i + 2\rho_i\nu_{ei}\frac{\partial w_i}{\partial z} = \tau_{zz,i+1} = -p_{i+1},\ \tau_{xz,i} = \rho_{ice}\nu_{ei}\left(\frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x}\right) = 0,\ z = -h_i,\ i = 1,\ 3. \tag{13}$$

Kinematic condition at the air–ice interface

$$w_i = \frac{\partial\eta_i}{\partial t},\ z = 0,\ i = 1,\ 3. \tag{14}$$

Continuity of velocity at the ice–water interface

$$w_i = w_{i+1} = \frac{\partial\eta_{i+1}}{\partial t},\ z = -h_i,\ i = 1,\ 3. \tag{15}$$

No penetration condition at the sea floor

$$w_{i+1} = 0,\ z = -H,\ i = 1,\ 3. \tag{16}$$

In terms of the Fourier modes, the solutions are

$$\phi_i(x,\ z,\ t) = (A_i(n)\cosh k_i(n)z + B_i(n)\sinh k_i(n)z)e^{ik_i(n)x}e^{-i\omega t}, \tag{17}$$

$$\psi_i(x,\ z,\ t) = (C_i(n)\cosh\alpha_i(n)z + D_i(n)\sinh\alpha_i(n)z)e^{ik_i(n)x}e^{-i\omega t}, \tag{18}$$

for the ice region $i = 1, 3$, and

$$\phi_{i+1}(x,\ z,\ t) = E_i(n)\cosh k_i(n)(z+H)e^{ik_i(n)x}e^{-i\omega t}, \tag{19}$$

for the water region $i+1 = 2, 4$. In the above, $\alpha_i^2(n) = k_i^2(n) - i\omega/\nu_{ei}$, $i = 1, 3$ and n indicates the n-th mode, as the dispersion relation to be shown has solutions, each one is an admissible mode. The no penetration condition at the sea floor is automatically satisfied by the cosh term in the potential and stream functions.

As shown in Appendix B in Zhao and Shen (2013), these boundary conditions together yield a set of homogeneous equations for $A_i(n)$, $B_i(n)$, $C_i(n)$, and $D_i(n)$ as shown below,

Download English Version:

https://daneshyari.com/en/article/6388094

Download Persian Version:

https://daneshyari.com/article/6388094

Daneshyari.com