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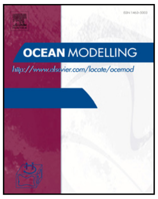

Parameterization of eddy fluxes based on a mesoscale energy budget

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ABSTRACT

It has recently been proposed to formulate eddy diffusivities in ocean models based on a mesoscale eddy kinetic energy (EKE) budget. Given an appropriate length scale, the mesoscale EKE can be used to estimate an eddy diffusivity based on mixing length theory. This paper discusses some of the open questions associated with the formulation of an EKE budget and mixing length, and proposes an improved energy budget-based parameterization for the mesoscale eddy diffusivity. A series of numerical simulations is performed, using an idealized flat-bottomed β-plane channel configuration with quadratic bottom drag. The results stress the importance of the mixing length formulation, as well as the formulation for the bottom signature of the mesoscale EKE, which is important in determining the rate of EKE dissipation. In the limit of vanishing planetary vorticity gradient, the mixing length is ultimately controlled by bottom drag, though the frictional arrest scale predicted by barotropic turbulence theory needs to be modified to account for the effects of baroclinicity. Any significant planetary vorticity gradient, β, is shown to suppress mixing, and limit the effective mixing length to the Rhines scale. While the EKE remains moderated by bottom friction, the bottom signature of EKE is shown to decrease as the appropriately non-dimensionalized friction increases, which considerably weakens the impact of changes in the bottom friction compared to barotropic turbulence. For moderate changes in the bottom-friction, eddy fluxes are thus reasonably well approximated by the scaling relation proposed by Held and Larichev (1996), which ignores the effect of bottom friction.

1. Introduction

The ocean circulation is strongly influenced by mesoscale turbulent eddies (e.g., Gill et al., 1974; Johnson and Bryden, 1989; Hallberg and Gnanadesikan, 2006; McWilliams, 2008; Waterman et al., 2011). However, the resolution of most current global ocean models is insufficient to resolve these eddies. Most current IPCC-class climate models use ocean components with typical horizontal resolutions of about one degree or coarser (Flato et al., 2013). Longer-term simulations, as used for paleo-climate applications, require even coarser grids, due to the prohibitive computational costs associated with long-term simulations at high resolution. At resolutions of about one degree or coarser, mesoscale eddies cannot be resolved, and their effects on the transport of tracers and physical properties must be parameterized (e.g., Hallberg and Gnanadesikan, 2006). Even when much higher resolutions are used and eddies are present in the tropics and subtropics, the effects of eddies will still need to be parameterized at higher latitudes and in near-coastal waters (Hallberg, 2013). Mesoscale eddy effects are typically parameterized with a tracer diffusion, which is strongly enhanced in the along-isopycnal direction (Redi, 1982), together with a closure based on Gent and McWilliams (1990) (hereafter: GM). The GM parameterization acts to flatten isopycnals by re-arranging water masses adiabatically. A closure of this form is motivated by the fact that eddies extract available potential energy stored in the mean flow, by rearranging water masses adiabatically (Gent et al., 1995). In an isopycnal layer model (which is naturally adiabatic) the GM parameterization can be described as a diffusion of the interface height between isopycnal layers (Gent et al., 1995; Vallis, 2006; Hallberg, 2013). A major question that remains is what sets the eddy tracer and interface height diffusivities. It is clear that both coefficients should vary in space and depend on properties of the resolved flow itself. Some dependence of the eddy diffusivity on the resolved flow is now commonly included in numerical ocean models (e.g., Farneti and Gent, 2011). However, exactly how this dependence should look remains unclear - yet it is of primary importance for the response of eddy transports to changes in the external forcing.

It has recently been proposed to formulate the eddy diffusivity based on a mesoscale eddy kinetic energy (EKE) budget (Cessi, 2008; Eden and Greatbatch, 2008; Marshall and Adcroft, 2010). Both the

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tracer and interface height eddy diffusivities are expected to scale with a typical eddy velocity times a mixing length. The eddy velocity can be inferred from the EKE, leaving the mixing length scale to be specified. A generally applicable scaling relation for the mixing length has not yet been derived. However, even assuming a constant mixing length, an EKE budget based parameterization may be expected to be superior to the assumption of a constant eddy diffusivity, as it takes into account the dependence of the eddy velocity on the mean state.

The goal of this paper is to build upon some of the arguments proposed by Cessi (2008) , Eden and Greatbatch (2008) and Marshall and Adcroft (2010). We will analyze a series of idealized numerical simulations, test some of the assumptions made in these previous studies, and discuss their implications for the estimated eddy diffusivity. Based on these considerations, we will propose an improved parameterization for the mesoscale eddy tracer diffusivity and GM transfer coefficient.

One focus here will be on the role of frictional dissipation. The EKE level in a statistical equilibrium is controlled by a balance between the net transfer of energy from the large-scale mean flow to EKE (by instabilities of the mean flow) and the dissipation of EKE. It has therefore been argued repeatedly that frictional dissipation must be important in controlling the level of EKE and with it the eddy diffusivity (e.g., Arbic and Flierl, 2004; Thompson and Young, 2007; Arbic and Scott, 2008; Cessi, 2008). However, none of the traditional parameterizations for the mesoscale eddy diffusivity (e.g., Green, 1970; Stone, 1972; Held and Larichev, 1996; Visbeck et al., 1997) includes any explicit dependence on parameters characterizing frictional dissipation. Of the EKE budget based arguments cited above, only Cessi (2008) and Marshall and Adcroft (2010) explicitly consider the role of frictional dissipation. In both cases frictional dissipation is described by a simple linear loss term in the EKE budget, seemingly consistent with the linear bottom drag assumed in the numerical simulations considered in these studies.

In addition to the role of frictional dissipation on the eddy energy budget, we will also make a new attempt at characterizing what sets the eddy mixing length. In the limit of vanishing planetary vorticity gradient and topography, the mixing length is ultimately limited by bottom friction. However, the frictional arrest scale predicted by barotropic turbulence theory (Grianik et al., 2004; Held, 1999) needs to be modified to include effects associated with baroclinicity. Moreover, any significant planetary vorticity gradient, β, is shown to suppress mixing, and limit the effective mixing length to the Rhines scale.

This paper focusses on some of the theoretical challenges in the formulation of the EKE budget and mixing length. A variant of the EKE budget equation is introduced in Section 2. In Section 3, we show results from a series of idealized numerical simulations, with the discussion focussing primarily on the mixing length, as well as the vertical structure of EKE - which controls the dissipation rate of EKE via bottom friction. In Section 4, we then use the results for the mixing length and vertical structure of EKE to derive a scaling relation for the eddy diffusivity, assuming a spatially and temporally local balance of EKE generation and dissipation (similar to Cessi, 2008). In Section 5, we discuss some outstanding questions and directions for future work, and we conclude with a summary of the main results in Section 6.

2. The EKE budget

Eden and Greatbatch (2008) formulate a predictive equation for the three-dimensional field of mesoscale eddy kinetic energy, for use in a numerical model which does not resolve the mesoscale flow. Such a local budget leaves some arbitrariness as to the exact formulation of large-scale to mesoscale energy transfer terms, with different formulations differing by flux terms, which vanish in a global integral, but may be large locally. Moreover, using any formulation, flux terms do arise and need to be parameterized. This provides a challenge in particular for the computation of the vertical structure of mesoscale EKE, which typically organizes mostly into the barotropic and lowest baroclinic modes (Wunsch, 1997). For simplicity, we here formulate a budget equation only for the total vertically integrated mesoscale EKE (as also done by Cessi, 2008), thus circumventing the need for an explicit parameterization of vertical EKE fluxes.

If we assume that the effect of mesoscale eddies on the large scale flow is represented by the GM parameterization and a viscous stress term, we can write the mesoscale EKE budget equation as

$$\partial_t E = \dot{E}_{GM} - \dot{E}_{fric} - \nabla \cdot \mathbf{T}. \tag{1}$$

$\dot{E}_{GM}$ is the energy loss of the large-scale flow associated with the GM parameterization - which parameterizes the conversion of large-scale available potential energy into mesoscale EKE by baroclinic instability. $\dot{E}_{fric}$ represents frictional dissipation of mesoscale EKE, and $\mathbf{T}$ denotes the horizontal transport of mesoscale EKE.

The simulations discussed in this paper employ an isopycnal layer model, in which the effect of the GM parameterization is obtained by a diffusion of the layer interface height, and thus

$$\dot{E}_{GM} = \frac{1}{H}\sum_i g_i' K_\eta |\nabla \overline{\eta_i}|^2 \tag{2}$$

where H is the total depth, g_i' is the reduced gravity at the ith layer interface, $\overline{\eta_i}$ is the interface height displacement of the large-scale "resolved" flow, and K_η is the interface height diffusivity, which is analog to the GM coefficient in a $z-$coordinate model (Gent et al., 1995; Vallis, 2006; Hallberg, 2013). The sum is here taken over all layer interfaces. As mentioned above, there is some freedom as to how exactly this term is formulated, with the difference between formulations amounting to a flux term, which vanishes upon global integration but not locally. The formulation in Eq. (2) has the desirable property that it is locally positive definite.

We don't include a direct transfer of kinetic energy between the large-scale resolved flow and the mesoscale eddies. A direct transfer from large-scale KE to mesoscale EKE represents the source of EKE in barotropic instability (e.g. Marshall and Adcroft, 2010), which, however, is not expected to be important in the simulations discussed below. In general, the sign of the net kinetic energy transfer between the large-scale flow and mesoscale eddies remains unclear. Jansen and Held (2014) propose to include a "backscatter" of KE from subgrid scales to the resolved flow in eddy permitting models, to represent the up-scale transfer of EKE in geostrophic turbulence. Even at coarser, non-eddying, resolution such a backscatter term (which can drive jets and Taylor caps) may be of at least similar magnitude to the potential source of mesoscale EKE associated with barotropic instability. For simplicity neither barotropic instability nor energy backscatter are included in the EKE budgets discussed here.

$\dot{E}_{fric}$ is the frictional dissipation of EKE. Unlike three-dimensional isotropic turbulence, geostrophic turbulence does generally not exhibit a direct kinetic energy cascade towards the micro-scale, where energy can be dissipated effectively by the molecular viscosity. This lack of a direct EKE cascade opens up the question of how mesoscale EKE is dissipated in the ocean. While the exact pathways of mesoscale EKE to dissipation remain unknown and heavily debated (e.g. Wunsch and Ferrari, 2004; Ferrari and Wunsch, 2009), there is both observational and numerical evidence for strongly enhanced dissipation near the bottom boundary, where rough topography generates energy transfers into internal waves and boundary layer turbulence (e.g. Ledwell et al., 2000; Nikurashin et al., 2013). In the numerical simulations discussed in this study frictional dissipation near the bottom boundary is parameterized using a quadratic bottom drag. A quadratic drag law follows directly from dimensional considerations, and is widely used in numerical ocean models (Egbert et al., 2004; Gill, 1982; Willebrand et al., 2001). Consistent with our numerical model we formulate the frictional dissipation of