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# Optimal dispersion with minimized Poisson equations for non-hydrostatic free surface flows

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ABSTRACT

A non-hydrostatic shallow-water model is proposed to simulate the wave propagation in situations where the ratio of the wave length to the water depth is small. It exploits the reduced-size stencil in the Poisson pressure solver to make the model less expensive in terms of memory and CPU time. We refer to this new technique as the minimized Poisson equations formulation. In the simplest case when the method applied to a two-layer model, the new model requires the same computational effort as depth-integrated non-hydrostatic models, but can provide a much better description of dispersive waves. To allow an easy implementation of the new method in depth-integrated models, the governing equations are transformed into a depth-integrated system, in which the velocity difference serves as an extra variable. The non-hydrostatic shallow-water model with minimized Poisson equations formulation produces good results in a series of numerical experiments, including a standing wave in a basin, a non-linear wave test, solitary wave propagation in a channel and a wave propagation over a submerged bar.

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## 1. Introduction

In fluid dynamics, dispersion means that waves of different wavelengths travel at different phase speeds. For surface gravity waves it plays an important role in wave transformation from deep water to intermediate water and in wave interactions with an uneven bottom. In the past decades, significant effort has been devoted to the development of models that can accurately and efficiently predict free surface wave propagation in a wide range of water depths (Marshall et al., 1997; Casulli and Stelling, 1998; Stansby and Zhou, 1998; Nwogu, 1993).

There are two conventional approaches for modeling the dispersive effects: the Boussinesq-type approach and non-hydrostatic models. Boussinesq-type equations (Peregrine, 1967) have provided a general framework to extend the applicability of depth-integrated equations into deep water. They are typically based on shallow water equations and they utilize an expansion in $kd$ ($k$ is the wave number and $d$ is the water depth).

The range of applicability of the conventional Boussinesq equations is limited to $kd < 0.75$, as stated in Madsen et al. (2002, 2003). Substantial effort has been devoted to extending the linear and nonlinear range of applicability of Boussinesq-type models. As a result, a number of enhanced and higher-order Boussinesq models have been developed. For instance, Nwogu (1993) used the velocity at an arbitrary distance from the still water level as the velocity variable and made the model applicable up to $kd \approx 3$. Using a fourth-order polynomial, Gobbi et al. (2000) developed a model which has good linear dispersive accuracy up to $kd \approx 6$. A considerable improvement by Madsen and Sørensen (1992) has resulted in a formulation including fifth-derivative operators, accurate to extremely deep water ($kd \approx 40$). Recently, Lynett and Liu (2004a) have also proposed a two-layer Boussinesq approach, with good linear wave characteristics up to $kd \approx 6$. They also extended this approach to multiple layers, and have achieved accurate linear dispersive properties up to $kd \approx 17$ with three layers, up to $kd \approx 30$ with four layers, including only third-order spatial derivatives (Lynett and Liu, 2004b). The principle behind Boussinesq formulations is to incorporate the effects of non-hydrostatic pressure, while eliminating the vertical coordinate. The high accuracy is at the expense of the simplicity and efficiency of the model. It results in a rather complicated system with high-order derivatives, which requires an equally complex numerical scheme and leads to instability over complex terrain.

The second approach is the non-hydrostatic models. In the original work of Chorin (1968), the so called 'projection method' was developed to solve the Navier–Stokes equations. The problem-solving process is split into two steps. In the first step, the velocity field is calculated by using the momentum equations without taking the pressure gradient into account. In the second step, the

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projection step, the resulting intermediate velocity field is then projected onto a divergence-free space by solving a Poisson equation of the full pressure, which is computationally expensive. To improve the efficiency of this method in the non-hydrostatic models, Marshall et al. (1997) and Casulli and Stelling (1998) independently proposed an alternative approach in which the pressure is decomposed into a hydrostatic component and a non-hydrostatic component. This pressure decomposition method has been successfully used in many non-hydrostatic models (such as MITgcm Marshall et al., 1997, Delft3D Bijvelds, 2003, SUNTANS Fringer et al., 2006, non-hydrostatic ROMS Kanarska et al., 2007 and SWASH Zijlema et al., 2011).

However, the implementation of the zero pressure boundary condition at the free-surface is difficult for a staggered grid where the pressure is located at the center of the cell. It has been recognized that 10–20 vertical layers are normally required in a staggered grid model to describe wave dispersion characteristics up to an acceptable level if the hydrostatic assumption is employed at the top layer (Casulli, 1999). To address the issue mentioned above, Stelling and Zijlema (2003) developed an efficient and accurate numerical method which utilizes a Keller-box scheme and an edge-based grid system in the vertical direction. This enables the non-hydrostatic pressure to be located at the cell faces rather than at the cell centers. Therefore, the top-layer pressure boundary condition can be assigned exactly without any approximation. Their model can resolve the frequency dispersion up to an acceptable level of accuracy with a small number of vertical layers.

Studies show that the dispersion property of non-hydrostatic models can be improved further by using more layers, without increasing the order of the spatial derivatives (Stelling and Zijlema, 2003; Yuan and Wu, 2006). However, the price to pay for such an improvement is a significant increase in computational cost, due to the large size of the resulting matrix system that needs to be solved. With the increasing demands of performing large scale flow simulations with non-hydrostatic models, improving their efficiency has become a competitive necessity. One of the promising directions in non-hydrostatic modeling is the use of a small number of vertical layers to efficiently and accurately model free-surface waves (Stelling and Zijlema, 2003; Yuan and Wu, 2004). Already in 2002, Reeuwijk (2002) proposed a method to improve the efficiency of non-hydrostatic models in which the number of pressure layers can be chosen independently from the number of horizontal velocity layers. A few free parameters were introduced to express the pressure at the place where the value of pressure is missing. However, in his model, the horizontal velocities are not independent variables. It is the summation of the horizontal velocities within each pressure layer servers as an independent variable in determining the dispersion relation. Increasing the number of horizontal momentum layers does not improve the dispersion accuracy which only depends on the number of pressure layers. This explains why the wave propagation speed in his computations is only determined by the number of pressure layers.

A similar idea to that described in Reeuwijk (2002) was recently presented, Bai and Cheung (2012b) proposed a parameterized non-hydrostatic pressure distribution to reduce the computational costs. A free parameter is introduced to express the non-hydrostatic pressure at the mid flow depth in terms of the bottom pressure. With this approximation, the two-layer flow system is reduced to a hybrid system with a free parameter. The free parameter is optimized against the exact linear dispersion relation in the range of $0 < kd < 3$. The computational cost is reduced to the same cost as that of a one-layer system. However, the accuracy of the dispersion is only improved slightly. The reason for this is the same as stated above for Reeuwijk (2002)'s model. Compared to a one-layer model, the hybrid system does not increase the freedom of the independent variables in determining the dispersion relation.

In this paper, an alternative approach is introduced to improve the accuracy of two-layer non-hydrostatic models without losing performance. It exploits the reduced-size stencil in the pressure Poisson equation. A simple stencil of the horizontal velocity in the bottom layer is proposed. With this simplified stencil and manipulations of the equations in the two bottom layers, the vertical velocity and pressure at the bottom can be eliminated from the system. The rank of the Poisson equation is reduced. Since most of the computational effort is devoted to inverting the Poisson matrix, reducing the dimension of the Poisson matrix leads to less computational cost.

The new method is referred to as the minimized Poisson equations formulation. It brings a considerable improvement to the method described in Stelling and Zijlema (2003). The governing equations have also been transformed into an equivalent, depth-integrated system, with the velocity difference as a correction to the depth-integrated flow. This allows an easy implementation of the method in depth-integrated models. The depth-integrated formulation is implemented in the newly developed, depth-integrated, two-dimensional, unstructured, non-hydrostatic finite volume model, $H_2$Ocean (Cui et al., 2010, 2012). For the same computational cost, the new model can achieve much more accurate linear dispersion than the one-layer model. Several classic test cases are used to validate the model. It is demonstrated that the new method leads to a significant reduction of computational effort, while maintaining high linear dispersion accuracy.

This paper proceeds as follows. In Section 2, the basic governing equations are presented. In Section 3, the minimized Poisson equations formulation is introduced. In Section 4, the method is applied to a two-layer system and is referred to as the reduced two-layer model. In Section 5, the reduced two-layer model has been transformed to a depth-integrated system to allow an easy implementation in one-layer non-hydrostatic models. Several test cases are given in Section 6 to demonstrate the accuracy and efficiency of the new formulation. Finally, in Section 7 the method is discussed.

## 2. Governing equations

The governing equations are the Euler equations with the pressure decomposed into hydrostatic ($p_h$) and non-hydrostatic components ($q$) using the notation in Casulli and Stelling (1998). Only the two-dimensional $(x, z)$ plane is considered here. The continuity equation and momentum equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + g\frac{\partial \eta}{\partial x} + \frac{\partial q}{\partial x} = 0 \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{\partial q}{\partial z} = 0 \tag{3}$$

where $\eta$ is the free surface elevation, $u$ and $w$ are the velocities in Cartesian coordinate system $(x, z)$, $q$ is the non-hydrostatic pressure, $p = p_h + q$ and $p_h = \rho_0 g(\eta - z)$ (see Fig. 1).

The kinematic boundary conditions at the free surface and at the bottom are given by

$$w = \frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} \quad \text{at} \quad z = \eta(x, t) \tag{4}$$

$$w = -u\frac{\partial d}{\partial x} \quad \text{at} \quad z = -d(x) \tag{5}$$

where $d$ is the still water depth. At the water surface, the non-hydrostatic pressure vanishes, $q|_{z=\eta} = 0$. Integrating Eq. (1) over

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