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Optimisation of an idealised ocean model, stochastic parameterisation of sub-grid eddies

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ABSTRACT

An optimisation scheme is developed to accurately represent the sub-grid scale forcing of a high dimensional chaotic ocean system. Using a simple parameterisation scheme, the velocity components of a 30 km resolution shallow water ocean model are optimised to have the same climatological mean and variance as that of a less viscous 7.5 km resolution model. The 5 day lag-covariance is also optimised, leading to a more accurate estimate of the high resolution response to forcing using the low resolution model.

The system considered is an idealised barotropic double gyre that is chaotic at both resolutions. Using the optimisation scheme, we find and apply the constant in time, but spatially varying, forcing term that is equal to the time integrated forcing of the sub-grid scale eddies. A linear stochastic term, independent of the large-scale flow, with no spatial correlation but a spatially varying amplitude and time scale is used to represent the transient eddies. The climatological mean, variance and 5 day lag-covariance of the velocity from a single high resolution integration is used to provide an optimisation target. No other high resolution statistics are required. Additional programming effort, for example to build a tangent linear or adjoint model, is not required either.

The focus of this paper is on the optimisation scheme and the accuracy of the optimised flow. However the forcing can provide insights in the design of deterministic and stochastic parameterisations. In the present study, we found that the stochastic parameterisation correcting the model variance is associated with the spatial pattern of eddy-decorrelation timescales rather than the spatial pattern of the amplitude of the variance. The method can be applied in future investigations into the physical processes that govern barotropic turbulence and it can perhaps be applied to help understand and correct biases in the mean and variance of a more realistic coarse or eddy-permitting ocean model. The method is complementary to current parameterisations and can be applied at the same time without modification.

1. Introduction

Due to the limitations of finite computational power, current numerical methods are not capable of accurately resolving the ocean circulation down to the viscous scale. Since there exists no universal sub-grid scale turbulence model that can close for all unresolved quantities (Reynolds stresses, turbulent fluxes, etc.) ad hoc representations are required, and state of the art numerical models exhibit serious differences and inaccuracies in their climatologies (e.g. Flato et al. (2013, Section 9.4.2)). The simplest approach to parameterise sub-grid scale processes is to dissipate any small-scale motion while simultaneously stabilising the model. This is typically achieved by employing an eddy diffusivity designed, for example, to improve spectral characteristics near the grid-scale (e.g. Smagorinsky (1963), Leith (1967)), or by using a diffusive integration scheme (e.g. Ritchie (1988)). Another approach is to mimic the physical processes in the real ocean. For example, mesoscale eddies in the ocean interior tend to rearrange fluid parcels along isopycnals (constant density surfaces) which leads to the widely implemented Gent–McWilliams parameterisation scheme in the tracer equations (Gent et al., 1990). Such approaches to find the sub-grid momentum or buoyancy forcing are often based upon the time-mean effect of the sub-grid scale forcing upon the large scale flow as diagnosed by comparing a low resolution model with measurements, or a high resolution integration. The approximate functional form of the sub-grid momentum or buoyancy forcing in terms of the grid scale flow of a turbulent system may be found using high resolution integrations (e.g. Achatz and Branstator (1999)), using, for example, a polynomial fit. A stochastic term

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may be used to represent the fit residual (e.g. Wilks (2005), Frederiksen and Kepert (2006), Zidikheri and Frederiksen (2009), Kitsios et al. (2013), Arnold et al. (2013)) or realistic variance (e.g. Hasselmann (1976), Buizza et al. (1999), Palmer (2001), Berloff (2005)). The deterministic and stochastic sub-grid forcing can be derived from theoretical considerations (e.g. Kraichnan (1959), Herring and Kraichnan (1972), Frederiksen and Davies (1997), Holm et al. (1999), Marshall et al. (2012), Grooms and Majda (2013), Mana and Zanna (2014)), although such an approach can be practically difficult to implement (Foiaş et al., 2001; Mana and Zanna, 2014).

In many cases one or more parameters that govern the strength of these schemes must be chosen with limited guidance from theory. Trial and error comparison of model output, as a function of parameter values, with ocean data, is one method often referred to as "tuning". Tuning up to around five or six parameters is possible in principle with a simple parameter search, however larger numbers of parameters require an "expert" opinion, independence from each other, or an acceptance that the optimal values will not be found. The problem is that to explore each direction of a parameter space of dimensionality d across n different climatologies requires n^d points to be evaluated. Given a high ($d \gg 5$) dimensional vector $\mathbf{p}$ of parameters, evaluation of the entire parameter space is not practical and in order to optimise anything we are forced to define an objective target to optimise for, or in other words a cost function $G(\mathbf{p})$ to minimise. From an initial guess $\mathbf{p}_0$ a direction to change $\mathbf{p}$ may be given by the gradient of the cost function

$$\mathbf{p}_1 = \mathbf{p}_0 - \left.\frac{\partial G(\mathbf{p})}{\partial \mathbf{p}}\right|_{\mathbf{p}=\mathbf{p}_0} \delta p. \tag{1}$$

Here $\mathbf{p}_1$ is an improved estimate of the optimal parameters in comparison with $\mathbf{p}_0$ and δp is a small positive constant with the appropriate units. The process can be iterated until no further optimisation is possible. Accurate estimation of $\partial G/\partial \mathbf{p}$ can be difficult, requiring for example tangent linear and adjoint models to be integrated. Implementation of adjoint models for ocean circulation problems has been achieved for sensitivity analysis and data assimilation capabilities (e.g. Marotzke et al. (1999), Moore et al. (2004)) as has optimisation of the eddy-buoyancy sub-grid parameters from the climatological mean state (Ferreira et al., 2005). However there are still some unresolved issues for large-scale chaotic systems. Firstly the programming effort is substantial leading to the development of semi-automatic differentiation packages for this purpose (e.g. Giering (1999), Heimbach et al. (2005)). Secondly if a system has a stochastic element the problem of optimising stochastic parameterisation has not, to the author's knowledge, been considered. Finally, although the adjoint approach is useful for short time optimisation in ocean (e.g. Gebbie et al. (2006), Mazloff et al. (2010), Balmaseda et al. (2013)) and atmosphere (e.g. Kalnay et al. (1996), Dee et al. (2011)) state estimation, it is not currently capable of optimising for the long time climate averages of a chaotic system (e.g. Lea et al. (2000), Eyink et al. (2004)) and approximations are required. Some attempts to solve this problem in a slightly different context include the methods of Abramov and Majda (2009) applied to climate response, who use the full non-linear model for the short time gradient estimate and a Gaussian model approximation for longer times, and Wang et al. (2014) who uses a modified adjoint algorithm to stabilise the gradient estimation algorithm. Fortunately an estimate of $\partial G/\partial \mathbf{p}$ does not need to be particularly accurate for the purposes of optimisation. It is merely required to follow a trajectory in parameter space that eventually leads toward the optimum and to tend to zero as the optimum is approached. Therefore we have the opportunity to optimise with a much simpler criteria if a very approximate direction of $\partial G/\partial \mathbf{p}$ can be found. This is the approach of the present paper. In our case, with the climate change problem in mind, the goal is accurate optimisation of the climatological mean and variance and approximate optimisation of the response of the system to a forcing, using a "truth" as the optimisation target.

1.1. The mean

Current state of the art ocean models exhibit a different climatological mean state to that observed in the real ocean (Flato et al., 2013). For example, the poor representation of eddy-mean flow processes leads to unrealistic western boundary currents (Gulf Stream and Kuroshio) responsible for large sea surface temperature biases (Large and Danabasoglu, 2006). Their predictions are therefore approximations about a different climatological mean point in state space to that of reality. To account for such deviations from the observed climatology, post integration bias correction is sometimes applied (e.g. Stockdale (1997)). A more accurate approach would be to have a model that has the correct climatological mean state in the first place. This can be achieved for example by adding a spatially varying, but constant in time, parameter to the right hand side of the governing equations (Achatz and Branstator, 1999). This spatially-varying time-independent parameter represents the contribution to the climatological mean of all of the sub-grid processes that are not included in the basic low resolution model minus any biases introduced by incorrect additional terms, such as high viscosity. The size of the improvement in accuracy relative to post integration bias correction can be important. For example, in a coupled ocean–atmosphere model some studies suggest that the mean location of the ocean boundary currents have an important impact upon atmospheric dynamics (e.g. Kirtman and Vecchi (2011), Scaife et al. (2011)). The ocean bias therefore has the potential to cause atmospheric bias that may be difficult to correct post integration.

1.2. The variance

Often, due to artificially high viscosity in a dynamical ocean model and the lack of sub-grid variability, the variance of the prognostic variables is underestimated. Without a time dependent external forcing such as the seasonal cycle, one can often obtain a steady state in very low-resolution ocean models, where time derivatives of all prognostic variables are equal to zero. In non-eddying ocean models, any effect of the variance due to eddies is therefore reduced or missing. The fluctuations brought about by resolving the eddies in an ocean model can potentially lead to additional dynamical regimes being explored (e.g. Palmer (2001), Palmer and Weisheimer (2011)) and important processes such as eddy saturation (Straub, 1993; Munday et al., 2013) or jet rectification (Berloff, 2005; Waterman et al., 2012; Waterman and Hoskins, 2013). In addition, the lack of variance between the members of an ensemble of model integrations contributes to over confidence, in a statistical sense, in model predictions. For these reasons we consider it desirable for our model climatological variance, and hence the turbulent eddy kinetic energy, to be as close as possible to the measured ocean variance. Moreover, since the correlations of a turbulent system decay in time, we would like the correlations of any parameterised source of variance to also decay after some time. The simplest approach is to add a stochastic term, with a spatially varying amplitude and time scale, to the right hand side of the governing equations. In this paper we require that the parameters governing such a process ensure that the model's climatological variance is as accurate as possible, relative to the "truth".