Ocean Modelling 83 (2014) 1–10

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Energetics and mixing efficiency of lock-exchange flow

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ARTICLE INFO

Article history:
Received 29 August 2013
Received in revised form 8 August 2014
Accepted 13 August 2014
Available online 28 August 2014

Keywords:
Lock-exchange flow
Mixing efficiency
Mixing
Background potential energy

ABSTRACT

Mixing efficiency between different water masses is typically assumed to be constant in diapycnal mixing parameterizations used in ocean models. As of now, most coarse resolution ocean circulation models employ a constant mixing efficiency value of 0.2 for the shear driven mixing, internal waves and bottom boundary layer parameterizations. This study investigates the energetics and mixing efficiency of the lock-exchange flow at different Reynolds numbers. The lock-exchange experiment resolves Kelvin–Helmholtz vortices and is an idealized test case for oceanic gravity currents. At first, the required spatial resolution for the direct numerical simulations (DNS) is determined in simulations at a constant Reynolds number of 3500. The evolution of background potential energy and tracer variance are used to assess model results. We found that the model spatial resolution should resolve at least the Kolmogorov scale but not necessarily the Batchelor scale if convergences of background potential energy, tracer variance and dissipation are considered. Simulations at Reynolds number of 125, 500, 1000, 2500, 3500, 5000, 6000, 10,000 show that the mixing efficiency in the lock-exchange flow is smaller than 0.2, and it saturates around 0.12 when Reynolds numbers exceed the value of 2500.

1. Introduction

Mixing is an irreversible process in which energy is extracted from the mean flow through vertical shear production and destabilizing buoyancy flux (Cushman-Roisin and Beckers, 2011). Turbulent mixing of water masses of different densities is an important process for both coastal and large-scale ocean circulation. Near oceanic coasts, mixing is an important consequence of the breaking of internal waves (Klymak et al., 2012). In the deep oceans, vertical stratification is maintained by irreversible mixing (Munk, 1966). The total amounts of mixing and available potential energy required to mix oceanic water masses are poorly known in the ocean. Only some estimates are available (Wunsch and Ferrari, 2004). A fundamental parameter needed to estimate mixing is the mixing efficiency coefficient. One possible definition of mixing efficiency is the ratio of irreversible mixing to the sum of irreversible mixing and kinetic energy dissipation (Peltier and Caulfield, 2003). The potential energy increase due to mixing processes is in fact bounded by the product of power input (i.e. winds and tides) and the mixing efficiency coefficient (Wunsch and Ferrari, 2004).

Mixing efficiency values are currently debated. One line of thought assumes that the mixing efficiency in turbulent flows is approximately constant at 0.2 (Osborn, 1980; Peters et al., 1995) with a definition of ratio of buoyancy flux to turbulent energy dissipation rate. Although this quantity has been used extensively by the oceanography community to compute mixing in the ocean (Gregg and Ozsoy, 2002; Peters and Johns, 2005), it conflicts with the historical definition of efficiency as pointed out by Moum (1996). The definition of "flux coefficient" (Γ) term by Osborn (1980) might be more appropriate to use instead of mixing efficiency. One can compute the mixing efficiency from flux coefficient as $\mu = \Gamma/(1+\Gamma) \approx 0.166$. Nevertheless the assumption of constant 0.2 mixing efficiency is consistent with the traditional view on the ocean circulation as described in the following. Sandström (1908) postulated that an overturning circulation cannot be maintained in a closed domain if the buoyancy sources and sinks are at the same geopotential level. This has fundamental implications for ocean circulation since the ocean is both cooled and heated at the surface (i.e. horizontal convection) (Ilıcak and Vallis, 2012). After Sandström's experiments, it was concluded that the ocean cannot sustain a meridional overturning circulation (MOC) if forced by buoyancy alone. An additional mechanical forcing is required to maintain the MOC. Energy inputs from surface winds and tides are the main candidates for this. Wunsch and Ferrari (2004) estimated that 2 TW of mechanical energy is required to maintain the MOC and that 1.5 TW of the total energy is coming from internal tides. Most climate models employ internal tides and bottom boundary layer parameterizations using the constant

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http://dx.doi.org/10.1016/j.ocemod.2014.08.003
1463-5003/

mixing efficiency of 0.2 (Simmons et al., 2004; Legg et al., 2006; Dunne et al., 2012).

Another line of thought puts the 0.2 value under discussion. Recent three-dimensional (3D) direct numerical simulations (DNS) show that horizontal convection can have strong overturning and be highly efficient (Scotti and White, 2011). As a result, mixing efficiency values between 0.8 and 0.9 are estimated (Scotti and White, 2011; Gayen et al., 2013). These values necessitate a change in our understanding of the ocean energy budget. If buoyancy forcing at the surface can be responsible for strong overturning and the sinking regions are so efficient, this means that the overall contribution of the other mechanical forcing mechanisms might be overestimated in the traditional view.

There is more evidence which disagrees with the traditional assumption of mixing efficiency being equal 0.2. Laboratory experiments of two-layer exchange flows spanned a wide range of Reynolds numbers (Re) up to 220,000 showed that the efficiency of mixing saturates with a value around 0.11 for $\mathrm{Re} > 50{,}000$ (Prastowo et al., 2008; Prastowo et al., 2009). Laboratory internal-wave breaking experiments found that mixing efficiency is between 0.03 and 0.08 for waves of varied incident amplitudes (Hult et al., 2011). There is also a wide spread of mixing efficiency values in observations. Ruddick et al. (1997) found that mixing efficiency decreases systematically with increasing density ratio and increases systematically with increasing buoyancy Reynolds number in the North Atlantic Central Water. They also described previous values of efficiency of mixing from a variety of locations by other observers (see their Table 2). These mixing efficiency values are ranging from 0 to 0.4 (Ruddick et al., 1997). In the numerical simulations of Caulfield and Peltier (2000) and Peltier and Caulfield (2003), the evolution of a Kelvin–Helmholtz roll during a transition from a 2D symmetric laminar flow to a fully 3D turbulent state is investigated. The simulations show that the instantaneous mixing efficiency is much larger than 0.2, but it decreases with time and the average mixing efficiency approaches a value of 0.15. In similar simulations, Mashayek and Peltier (2011) found that the time-averaged efficiency increases with increasing Re and the mixing efficiency reaches values as large as 0.5. In addition, Mashayek et al. (2013) employed direct numerical simulations of shear-induced turbulence in stably stratified free shear flow. They also showed that constant 0.2 mixing efficiency fails at higher Richardson numbers provided that the Reynolds number is sufficiently high. Smyth et al. (2001) investigated the time evolution of mixing in turbulent overturns using both numerical simulations and microstructure profiles obtained during field experiments. They showed that mixing efficiency can change by more than an order of magnitude over the life of a turbulent overturn. Pham and Sarkar (2010) investigated the interaction between an unstable shear layer and a stably stratified jet showing that mixing efficiency exceeds 0.2, especially in highly turbulent areas.

The aim of this study is to investigate the mixing efficiency values in idealized gravity current simulations. A number of 3D direct numerical simulations of lock-exchange flow are performed. Such a choice is justified by the fact that the lock-exchange experiment is highly relevant to gravity currents, overflows (Ilıcak et al., 2008) and exchange flows in the ocean (Ilıcak et al., 2009; Ilıcak and Armi, 2010). In each of these shear- and buoyancy-driven flows, mixing occurs through Kelvin–Helmholtz instabilities. The advantage of the 3D lock-exchange problem is that it contains different turbulent processes such as shear-driven mixing, internal waves and gravitationally-unstable phases in an enclosed domain (Özgökmen et al., 2009). The effect of Reynolds number on the mixing efficiency is investigated. Different simulations are conducted spanning a wide range of Re from 125 and to 10,000. Mixing of the density field is quantified using the background potential energy (Winters et al., 1995). Results indicate that the mixing increases (i.e. background potential energy increases) as the Reynolds number increases. For the low and moderate Reynolds numbers, the mixing increases monotonically. At $\mathrm{Re} = 2500$, the mixing efficiency saturates at around 0.12. All mixing efficiency values are, however, consistently lower than 0.2.

The paper is organized as follows: In Section 2, the numerical model and the parameters of all numerical simulations are described. Results for the lock-exchange problem are presented in Section 3. Finally, major findings are summarized in Section 4.

2. Model description and numerical setup

In this study, the non-hydrostatic version of MIT general circulation model (MITgcm) is used with the Boussinesq approximation. The MITgcm is a three dimensional C-grid fully incompressible Navier Stokes equations model (Marshall et al., 1997). The non-dimensional model governing equations are

$$\frac{D\mathbf{u_i}}{Dt} = -\frac{\partial p}{\partial x_\mathbf{i}} - Ri_0 \rho' \delta_{i3} + \frac{1}{\mathrm{Re}} \frac{\partial^2 \mathbf{u_i}}{\partial x_j^2}, \tag{1}$$

$$\frac{\partial \mathbf{u}_j}{\partial x_j} = 0, \tag{2}$$

$$\frac{D\rho'}{Dt} = \frac{1}{\mathrm{RePr}} \frac{\partial^2 \rho'}{\partial x_j^2}, \tag{3}$$

where $\mathbf{u}$ is the three dimensional velocity, p is the pressure, ρ' is the density perturbation and D/Dt is the material derivative. The non-dimensionalization is performed using a characteristic velocity (U_0), a characteristic length scale (l_0), a characteristic time scale ($\tau^* = l_0/U_0$), density difference ($\Delta\rho'$) and pressure $\rho_0 U_0^2$ for $\mathbf{u}$, x, ρ', and p respectively. The initial Reynolds number ($Re = U_0 l_0/\nu$) expresses the relative importance of viscous effects where ν is the molecular viscosity. The bulk Richardson number ($Ri_0 = g\Delta\rho' 0.5H/(\rho_0 \Delta U_0^2)$) indicates the importance of stratification and shear. Finally, the Prandtl number is defined as $\mathrm{Pr} = \nu/\kappa$, where κ is the molecular diffusivity.

The non-dimensional computational domain is $0 \leqslant x \leq 8,\ 0 \leqslant y \leqslant 1/2$ and $0 \leqslant z \leqslant 1$ since vertical depth is used as the characteristic length scale, $H = l_0$. At all boundaries, no-flow and free-slip boundary conditions are used for the velocity components, while no-flux conditions are used for the density perturbation ρ'. The lock-exchange problem is initialized with dense fluid on the left separated from the light fluid on the right

Table 1
Numerical simulations setup and mixing efficiency values for Re = 3500. To resolve the Kolmogorov scale 94.2 million grid points are required.

$\mathrm{Re} = 3500$	# of grid points in x, y, z	Grid resolution (Δ)	Total # of points (N^3)	Mixing efficiency
Exp1	$576 \times 36 \times 72$	$\Delta = 1.38 \times 10^{-2}$	$\approx$ 1.5 million	0.1312
Exp2	$1152 \times 72 \times 144$	$\Delta = 6.94 \times 10^{-3}$	$\approx$ 12 million	0.1019
Exp3	$2304 \times 144 \times 288$	$\Delta = 3.47 \times 10^{-3}$	$\approx$ 95 million	0.0956
Exp4	$3456 \times 216 \times 432$	$\Delta = 2.31 \times 10^{-3}$	$\approx$ 322 million	0.0955