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A nonhydrostatic, isopycnal-coordinate ocean model for internal waves

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ABSTRACT

We present a nonhydrostatic ocean model with an isopycnal (density-following) vertical coordinate system. The primary motivation for the model is the proper treatment of nonhydrostatic dispersion and the formation of nonlinear internal solitary waves. The nonhydrostatic, isopycnal-coordinate formulation may be preferable to nonhydrostatic formulations in z- and σ-coordinates because it improves computational efficiency by reducing the number of vertical grid points and eliminates spurious diapycnal mixing and solitary-wave amplitude loss due to numerical diffusion of scalars. The model equations invoke a mild isopycnal-slope approximation to remove small metric terms associated with diffusion and nonhydrostatic pressure from the momentum equations and to reduce the pressure Poisson equation to a symmetric linear system. Avoiding this approximation requires a costlier inversion of a non-symmetric linear system. We demonstrate that the model is capable of simulating nonlinear internal solitary waves for simplified and physically-realistic ocean-scale problems with a reduced number of layers.

1. Introduction

1.1. Literature review

Simulating internal waves is one of the most computationally challenging tasks in ocean modeling. Large-scale processes such as internal tides can be modeled reasonably well with computationally-inexpensive hydrostatic models (Kantha and Clayson, 2000). Simulations of nonlinear internal solitary waves, on the other hand, require computationally-expensive nonhydrostatic models to represent dispersive behavior. Nonhydrostatic models can incur an order of magnitude increase in computational time relative to hydrostatic models due to the elliptic solver for the nonhydrostatic or dynamic pressure (Fringer et al., 2006). Furthermore, simulations of nonlinear internal solitary waves require high horizontal grid resolution to ensure that numerically-induced dispersion[1] is small relative to physical dispersion (Vitousek and Fringer, 2011).

The vertical coordinate system is often reported as the most important aspect in the design of an ocean model (Griffies et al., 2000; Chassignet et al., 2000; Willebrand et al., 2001; Chassignet, 2011). The three vertical coordinate systems typically used in ocean models are: (1) Height or z-coordinates, (2) Terrain-following or σ-coordinates, and (3) Isopycnal or ρ-coordinates. Each approach has numerous advantages and disadvantages as outlined in Griffies et al. (2000). Existing nonhydrostatic models employ z- or σ-coordinates [e.g. (Mahadevan et al., 1996a,b), MITgcm (Marshall et al., 1997b,a), SUNTANS (Fringer et al., 2006) for z-coordinates, POM (Kanarska and Maderich, 2003), BOM (Heggelund et al., 2004), ROMS (Kanarska et al., 2007), FVCOM (Lai et al., 2010a) for σ-coordinates and ICOM (Ford et al., 2004a,b) for vertically unstructured coordinates]. z- and σ-coordinate models are capable of representing overturning motions and eddies (e.g. Kelvin–Helmholtz, Rayleigh–Taylor, and other instabilities) that are associated with many small-scale nonhydrostatic processes. Isopycnal-coordinate models, on the other hand, cannot represent overturning motions or unstable stratification. This deficiency leads to the notion that isopycnal coordinates are not suitable for modeling nonhydrostatic processes (Adcroft and Hallberg, 2006). Consequently, existing isopycnal models such as MICOM/HYCOM (Bleck et al., 1992; Bleck, 2002), HIM (Hallberg, 1995, 1997; Hallberg and Rhines, 1996), POSEIDON (Schopf and Loughe, 1995), POSUM (Higdon and de Szoeke, 1997; de Szoeke, 2000) so far exclusively employ the hydrostatic approximation. While clearly a deficiency of isopycnal models, the inability to represent unstable stratification is an issue for hydrostatic and nonhydrostatic isopycnal formulations alike. In this paper, we do not propose a means for isopycnal-coordinate models to represent unstable stratification—this task is clearly suited to z- and σ-coordinate models. Instead, the primary motivation behind the model presented in this paper is the proper treatment

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[1] from truncation error of the discretized model equations

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of dispersion and the formation of nonlinear internal solitary waves in the context of an isopycnal-coordinate model. Internal solitary waves are clearly nonhydrostatic and not associated with overturning motions. Although overturning structures may exist in the vicinity of internal wave generation sites that might preclude the use of an isopycnal-coordinate model, a model is not required to resolve these structures to obtain a good prediction of the internal wave generation. For example, Klymak and Legg (2010) and Klymak et al. (2010) developed a simple scheme that faithfully captures dissipation and mixing related to internal wave generation at a ridge and show that their hydrostatic model was almost identical to the nonhydrostatic model in predicting the generation dynamics. This suggests that, while small-scale, nonhydrostatic processes related to internal wave generation are indeed complex, they can be parameterized appropriately in large-scale z-, σ-, and isopycnal-coordinate models that do not resolve them. Hence, a nonhydrostatic, isopycnal-coordinate formulation may be suitable for modeling internal or interfacial waves.

In the context of internal wave modeling, isopycnal-coordinates may provide some advantages over z- and σ-coordinates. Isopycnal or density-following coordinates provide natural representations of (stably) stratified fluids. This reduces the number of vertical grid points from $\mathcal{O}(100)$ in z- and σ-coordinate models to $\mathcal{O}(1)$ in isopycnal coordinates (Bleck and Boudra, 1981). Ideally, in locations where the internal wave structure is predominantly mode-1, an isopycnal model with only two layers may suffice (Simmons et al., 2011). The primary disadvantage of modeling nonhydrostatic pressure is that it requires solution of a three-dimensional elliptic (Poisson) equation for the nonhydrostatic pressure which significantly increases the computational cost relative to the hydrostatic model. Solving this elliptic equation requires optimally $\mathcal{O}(N)$ operations (Briggs et al., 2000) where N is the number of grid cells. Isopycnal coordinates can improve the efficiency of nonhydrostatic methods by reducing the required number of vertical grid points by an order of magnitude relative to z- and σ-coordinate models. Thus, reducing the number of vertical layers and thus the overall number of grid cells by an order of magnitude can result in at least one order of magnitude reduction in computational cost.

Another advantage of isopycnal-coordinates is the reduction or elimination of spurious diapycnal mixing. Transport in the ocean predominantly occurs along rather than across isopycnal surfaces (Iselin, 1939; Montgomery, 1940). In many applications, spurious diapycnal mixing, arising from numerically-diffusive truncation error in scalar transport schemes in z- and σ-coordinate models (Fringer et al., 2005), can be larger than physical diapycnal mixing (Griffies et al., 2000). Isopycnal coordinates, on the other hand, are not susceptible to spurious diapycnal mixing because the governing equations are constructed to directly control the amount of diapycnal transport – if any (Bleck and Boudra, 1981; Griffies et al., 2000). Hence, the problem of energy loss due to spurious diapycnal mixing that occurs in numerical models during the formation and propagation of internal solitary waves (Hodges et al., 2006) may be reduced or eliminated with isopycnal coordinates.

1.2. Outline of the proposed model

Existing approaches for simulating nonhydrostatic internal waves include z- and σ-coordinate models applied at high resolution in 3-D (Fringer et al., 2006; Vlasenko and Stashchuk, 2007; Vlasenko et al., 2009; Vlasenko et al., 2010; Lai et al., 2010b; Zhang et al., 2011; Guo et al., 2011) or 2-D slices (Scotti et al., 2007; Scotti et al., 2008; Buijsman et al., 2010) or asymptotic/Boussinesq-type approaches using 2-layer (Brandt et al., 1997; Choi and Camassa, 1999; Lynett and Liu, 2002; de la Fuente et al., 2008; Steinmoeller et al., 2012) or multi-layer models (Liu and Wang, 2012). Asymptotic or Boussinesq-type approaches do not require a pressure projection method. Instead, they include higher-order derivatives to account for the nonlinear and dispersive behavior. Boussinesq-type models have a limited range of applicability that is often the weakly nonlinear, weakly nonhydrostatic regime. To extend this range of applicability, more terms may be included or advanced formulations may be introduced. However, this can lead to an unwieldy set of governing equations containing high-order, mixed time-and-space derivatives.

The formulation presented here is intended to be flexible (using an arbitrary number of layers) and straightforward (resembling existing ocean models). The numerical method uses a pressure projection method which results in an elliptic equation for the dynamic pressure (as in z- and σ-coordinate models). The elliptic equation in isopycnal coordinates results in a non-symmetric system of linear equations. However, by invoking a mild-slope approximation, the system becomes symmetric and remarkably similar to the elliptic equation in z-coordinates.

Another significant difference between existing isopycnal models and the model presented here (besides the treatment of nonhydrostatic pressure) is the time-stepping procedure. Most isopycnal models use mode-splitting to treat fast free-surface gravity waves (Bleck and Smith, 1990; Higdon and Bennett, 1996; Higdon and de Szoeke, 1997; Hallberg, 1997). The current model uses an implicit time-stepping procedure for the free surface following Casulli (1999) which is common in nonhydrostatic models in z- and σ-coordinates (a list of nonhydrostatic models using implicit time-stepping procedures is given in Vitousek and Fringer (2013)). Casulli (1997) developed a hydrostatic, isopycnal model with an implicit time-stepping procedure for the free surface and layer heights. In his approach, the gradient of the Montgomery potential (M), which represents the hydrostatic pressure in isopycnal coordinates, is discretized implicitly. Thus, computing the free-surface and interface heights requires the inversion of a large (3-D) system of equations which is comparable in cost to the solution of the (3-D) elliptic equation for the nonhydrostatic pressure. The current model is similar to the approach of Casulli (1997). However, we split the Montgomery potential into barotropic ($M^{(bt)}$) and baroclinic ($M^{(bc)}$) parts according to

$$M = \rho_0^{-1}(p_h + \rho g z) = \underbrace{\rho_0^{-1}(p_s + \rho_0 g \eta)}_{=M^{(bt)}} + M^{(bc)}, \tag{1}$$

where ρ is the density (ρ_0 is the reference density), p_h is the hydrostatic pressure, p_s is the surface (atmospheric) pressure, η is the free-surface height, z is the interface location, and the term $\rho g z$ originates from the transformation to isopycnal coordinates. Because the barotropic and baroclinic portions of the Montgomery potential represent fast free-surface and slow internal-gravity waves, they are discretized implicitly and explicitly, respectively. This discretization requires inversion of a 2-D system in the horizontal to compute the free-surface height as is the case with implicit free-surface models in z- and σ-coordinates. The computational cost of the 2-D inversion for the free-surface height is minimal compared to the 3-D inversion for the nonhydrostatic pressure.

1.3. Use of Lagrangian coordinates for the nonhydrostatic equations

Adcroft and Hallberg (2006) conclude that the nonhydrostatic projection method and use of Lagrangian vertical coordinates are mutually exclusive. Their argument is based on how the Lagrangian algorithm prescribes the material derivative of the (general) vertical coordinate, $\dot{r}$, in the continuity equation while in the nonhydrostatic projection method this term should instead come from the vertical momentum equation. This leads to a conundrum in which one cannot simultaneously supply and diagnose a quantity in an equation (Adcroft and Hallberg, 2006). The present study does not seem lim-