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Modelling of partially-resolved oceanic symmetric instability

S.D. Bachman, J.R. Taylor *

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, United Kingdom

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ABSTRACT

A series of idealized numerical models have been developed to investigate the effects of partially resolved symmetric instability (SI) in oceanic general circulation models. An analysis of the energetics of symmetric instability is used to argue that the mixed layer can be at least partially restratified even when some SI modes are absent due to either large horizontal viscosity or coarse model resolution. Linear stability analysis reveals that in the idealized models the amount of restratification can be predicted as a function of the grid spacing and viscosity. The models themselves are used to demonstrate these predictions and reveal three possible outcomes in steady-state: (1) incomplete restratification due to viscosity, (2) incomplete restratification due to resolution, and (3) excessive restratification due to anisotropy of the viscosity. The third outcome occurs even on a high-resolution isotropic grid and in two separate numerical models, and thus appears to be a sort of robust numerical feature. The three outcomes are used to recommend criteria that a successful SI parameterization should satisfy.

1. Introduction

Regional ocean models are able to resolve smaller-scale features than are normally permitted by climate-scale GCMs. The oceanic submesoscale in particular is a popular topic of study in such models, due to its role as a "bridge" between the large-scale circulation and small-scale flows where mixing and dissipation can occur. Relatively little is known about the dynamics of submesoscale flows because of limitations in computational and observational resources (Capet et al., 2008a), but they are generally understood to have the following characteristics: (1) frontal structures are ubiquitous and are associated with potential and kinetic energy (Spall, 1995; Thomas and Ferrari, 2008; Thomas et al., 2008), (2) a variety of instabilities develop which feed off of the kinetic and/or potential energy and generate submesoscale motions (Mahadevan and Tandon, 2006; Mahadevan, 2006; Capet et al., 2008a,b,c; Fox-Kemper et al., 2008; Klein et al., 2008), (3) the Rossby (Ro) and Richardson (Ri) numbers are $\mathcal{O}(1)$, meaning that balanced models are not appropriate to describe the motion (Molemaker et al., 2005), and (4) submesoscales interact vigorously with other small-scale, high-frequency motions including Langmuir turbulence (Li et al., 2012; Van Roekel et al., 2012) and near-inertial waves (Whitt and Thomas, 2013; Joyce et al., 2013), thereby enhancing the downscale energy cascade.

The role of the submesoscale as an intermediate-scale bridge between the mean circulation and small-scale processes makes its study all the more important. Even in regional models, however, computational limitations affect how much of the submesoscale range can actually be represented in a model – a simulation run at coarse resolution inherently deemphasizes small-scale processes, and a fine-scale simulation with a smaller domain size may miss important interactions between the submesoscale and mesoscale flows. With respect to the small-scale processes, it is an open question as to what resolution is necessary to begin resolving certain types of submesoscale instabilities. The focus of this paper is on the resolvability of one such type of instability, namely symmetric instability (hereafter SI). Research on SI is at an early stage, and to the authors' knowledge no previous studies have systematically explored what resolution is required to resolve it in ocean models.

As computational power increases, models are able to simultaneously resolve a richer set of dynamics by running at higher spatial resolution and incorporating more complex physical and biogeochemical parameterizations. However, higher spatial resolution introduces a new set of challenges as well, the first among these being the issue of double-counting (Delworth et al., 2012). It is commonly thought that as models enter an "eddy-permitting" regime, where some (but not all) of the mesoscale eddies are explicitly resolved, parameterizations should either be turned off or minimized in order to prevent both resolving and parameterizing the same eddies. One reason for this is that parameterizations can out-compete the resolved eddies for the energy sources

* Corresponding author. Address: DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, United Kingdom. Tel.: +44 01223 337030.
E-mail address: J.R.Taylor@damtp.cam.ac.uk (J.R. Taylor).

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required to grow, leaving the resolved eddies weak and ineffectual (Henning and Vallis, 2004). Therefore, one of the first steps to developing a skillful parameterization is to know when its use is appropriate, and when it should be turned off to avoid double-counting.

The issue of double-counting is not confined to just mesoscale eddies, however. Submesoscales develop at scales less than 10 km, and these in turn will become partially resolved as GCM resolution becomes even finer in upcoming model generations. SI is one such submesoscale process, and ocean models will increasingly pass into a regime that could be described as "SI-permitting". As is the case with mesoscale eddies, explicitly resolving only some of the SI modes can be expected to present a challenge in preventing double-counting by a parameterization. As of the writing of this paper no parameterization exists for SI in the oceanic mixed layer, and any forthcoming attempt at one will require knowledge of how SI behaves when it is partially resolved.

Symmetric instability in a stably stratified flow occurs when the Ertel PV takes on the opposite sign of f (Hoskins, 1974). Fronts in the surface mixed layer of the ocean feature strong lateral density gradients, which in conjunction with wind forcing and/or buoyancy fluxes create conditions favorable to the development of SI (Thomas and Taylor, 2010). SI is capable of restratifying the mixed layer on timescales shorter than that of baroclinic instability (Haine and Marshall, 1998; Boccaletti et al., 2007; Li et al., 2012), and both types of instability are central to setting the stratification of the surface ocean at strong fronts.

Energetically, SI can be described as a small-scale shear instability that extracts energy from the vertically-sheared thermal wind (Taylor and Ferrari, 2010; Thomas and Taylor, 2010) and acts as a mediator in the dissipation of oceanic kinetic energy, helping to drive a forward cascade of energy from large to small scales. The term "mediator" is used here because the SI itself is not responsible for dissipation – its length scales are orders of magnitude larger than the dissipation scale, and so it relies on even smaller-scale turbulence to transfer energy downscale to be dissipated. Taylor and Ferrari (2009) showed that finite-amplitude SI develops secondary Kelvin–Helmholtz instabilities along bands of enhanced shear, which then break down into smaller-scale turbulence. However, Kelvin–Helmholtz instabilities are generally understood as 3D processes that are directly resolved in isotropic, very fine-scale simulations such as large-eddy simulations; aside from exceptional circumstances, they would not be resolvable in a regional model with a highly anisotropic grid. This introduces the related question of how and whether SI can restratify the mixed layer in a model when its associated secondary instabilities are not present?.

The objective of this paper is to investigate the level of spatial resolution necessary to explicitly resolve SI and to explore how the resolution threshold varies as a function of the mean flow parameters. The spatial scales at which models become SI-permitting are expected to also straddle the threshold between hydrostatic and non-hydrostatic flows; therefore, the resolution requirement is explored in both regimes. The discretization of the grid and the level of model viscosity can also affect the stability of the flow to SI, and so these possibilities are explored as well.

The main text that follows will be subdivided into two sections. The basic stability, energetics, and growth of SI will be discussed in Section 2. The differences between the growth of inviscid and viscously damped SI modes is shown, along with implications about what this may mean for the resolvability of SI in ocean models. Section 3 shows the results from a series of 2D simulations run at various resolutions, illustrating how the post-restratification character of the mixed layer can vary depending on the model viscosity and grid spacing. A summary of the main results and conclusions appears in Section 4. A detailed linear stability analysis of SI can be found in Appendix A.

2. Energetics of SI

The surface ocean is marked by the presence of sharp lateral density gradients formed as a result of frontogenesis. The presence of these lateral gradients modifies the turbulence that arises at the surface due in part to buoyancy loss (Haine and Marshall, 1998) and down-front wind stress (Thomas and Taylor, 2010), and introduces a variety of secondary effects that modulate buoyancy transport through the mixed layer (Thomas and Lee, 2005).

SI can be viewed as a hybrid of convective and inertial instabilities (Haine and Marshall, 1998). Since it is characterized by slantwise motions tilting across the lateral buoyancy gradient, SI is sometimes called "slantwise convection" (Emanuel, 1994). However, as pointed out by Thorpe and Rotunno (1989), SI has many features that are distinctly different from convection. For example, the most unstable motions are often aligned with isopycnals and are associated with a very small buoyancy flux. In fact, while convection is generated through a conversion of potential energy (PE) to kinetic energy (KE) by lowering the center of mass of the fluid, it is possible for SI to *raise* the center of mass and reduce the vertical stratification. Therefore, to avoid confusion, the term SI will be used rather than slantwise convection throughout the rest of this paper.

SI is one among a hierarchy of hydrodynamical instabilities thought to be prevalent in the ocean mixed layer. It is characterized by perturbations that are independent of the along-front direction. It also differs from baroclinic instability in that it can derive its energy by reducing the geostrophic shear via turbulent Reynolds stresses (Thomas et al., 2013) in addition to extracting PE from the background flow.

The growth of symmetric instability is best understood in terms of the Ertel potential vorticity (PV), which can be defined as

$$q = (f\mathbf{k} + \nabla \times \mathbf{u}) \cdot \nabla b, \tag{1}$$

where here the Coriolis parameter f is a constant under the f-plane approximation. Define the buoyancy frequency $N^2 = \partial b/dz$ and the horizontal buoyancy gradient $M^2 = \partial b/dx$, taking both to be constant but not necessarily equal to each other. Let the velocity field be $\mathbf{v} = V_B(x) + V_G(z)$, where V_B is a barotropic velocity and V_G the thermal wind velocity in balance with the lateral stratification, so that $dV_G/dz = M^2/f$. Furthermore, assume that the flow is homogeneous in the along-front direction y. The PV for this basic state is $q = (f + \zeta)N^2 - M^4/f$, where $\zeta = dV_B/dx$ is the relative vorticity, and can become negative for a sufficiently strong lateral buoyancy gradient. An alternative criteria for the growth of symmetric instability in such a balanced model is that the bulk Richardson number

$$Ri = \frac{N^2}{\left(\frac{dV_G}{dz}\right)^2} \equiv \frac{f^2 N^2}{M^4} \tag{2}$$

is such that

$$Ri < \frac{f}{f + \zeta} \quad \text{if} \quad f(f + \zeta) > 0. \tag{3}$$

Under these conditions SI is the most unstable mode when $0.25 < Ri < 0.95$ (Stone, 1966, 1970). The stratification throughout most of ocean interior is strong enough to render the flow stable to SI, with the notable exception of the surface and bottom boundary layers (Allen and Newberger, 1998). In the surface mixed layer, conditions for SI to grow are realized by surface forcing that destroys PV until regions of negative PV develop (Thomas, 2005). SI will then quickly restore the fluid to a marginally stable state (Thorpe and Rotunno, 1989) by mixing in fluid of higher PV from either the thermocline or the surface boundary layer. This mixing was discussed at length by Taylor and Ferrari (2009), who showed that SI locally enhances the shear to such an extent that secondary