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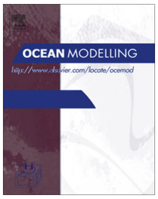

Ocean wave transmission and reflection between two connecting viscoelastic ice covers: An approximate solution

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A B S T R A C T

An approximate solution for wave transmission and reflection between open water and a viscoelastic ice cover was developed earlier, in which both the water and the ice cover were treated as a continuum, each governed by its own equation of motion. The interface conditions included matching velocity and stresses between the two continua. The analysis provided a first step towards modeling the wave-in-ice climate on a geophysical scale, where properties of the ice cover change with time and location. In this study, we derive the wave transmission and reflection from one viscoelastic material to another. Only two modes of the dispersion relation are considered and the horizontal boundary conditions are approximated by matching the mean values. The reflection and transmission coefficients are first determined for simplified cases to compare with earlier theories. All results show reasonable agreement when the same physical parameters are used. Behaviors of the transmission and reflection coefficients are then obtained for a range of viscoelastic covers. A mode switching phenomenon with increasing ice shear modulus is found. This phenomenon was pointed out in the study of wave propagation from open water to a viscoelastic cover. For two connecting viscoelastic covers, such mode switching is found to terminate with increasing viscosity. Together with an earlier investigation of wave dispersion in a viscoelastic ice cover, the present study provides a way to implement theoretical results in a numerical model for wave propagation through a heterogeneous ice cover. In discretizing a continuously changing ice cover over the geophysical scale, on top of the energy advection, energy transmission between computational cells due to the heterogeneity can be estimated using the present method, while the attenuation and wave speed within each cell are from the previously obtained dispersion relation. In addition, on floe scales, this study provides a way to determine wave scattering from an ice floe imbedded in grease or brash ice.

1. Introduction

As part of the climate change scenario, global wind and wave heights have increased in the past two decades (Young et al., 2011). In particular, wave conditions in the Arctic have intensified as the ice cover shrinks (Francis et al., 2011). The pressure for better wave models in the Arctic increases as the economic and environmental interests in this region rise. At present, existing operational wave models can only treat ice covers crudely. For example, in WAVEWATCH III, an ice cover is considered as a stepwise filter in such a way that the fraction of wave energy flux at any location varies linearly between 0 and 1, with two threshold values controlling this stepwise linear variation, both are related to the local ice concentration (Tolman, 2003). The group velocity is assumed unaltered from the open water condition. This model was established at a time when the only available ice parameter was the ice concentration and the wave conditions in the Arctic were not of great concern. In reality, waves can penetrate into ice-covered seas over a very long distance. Along its passage, wave energy is dissipated by the ice field. The attenuation rate depends on the wave period, ice concentration, thickness and floe size distribution (Wadhams et al., 1988; Squire et al., 1995; Squire, 2007). In turn, waves may break the ice floes and further complicate their interactive nature (Dumont et al., 2011). In addition, wind-wave generation may be modified greatly in the presence of a partial ice cover (Masson and Leblond, 1989; Perrie and Hu, 1996). Integrating ice effects into wave models will advance wave predictions in ice-covered seas. With better remote sensing capabilities, information on ice conditions will improve. Wave models that can utilize this improvement need to be developed.

In an earlier study, Wang and Shen (2010, 2011) proposed a linear viscoelastic model to represent a general ice cover. This continuum-based approach allowed the water and the overlying ice cover to each deform internally according to its own material properties. The requirement of matching interfacial conditions determine both the dispersion relation of the wave that travels into

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the ice cover (Wang and Shen, 2010) and the reflection and transmission of wave energy from open water into an ice cover (Wang and Shen, 2011). The dispersion relation was shown to converge to three types of existing models under respective limiting conditions: the mass loading (Peters, 1950; Weitz and Keller, 1950), thin elastic plate (Wadhams, 1973a), and pure viscous layer (Keller, 1998). For general viscoelastic ice layer of finite thickness, many coexisting modes for the transmitted wave were possible. The series of coexisting modes were truncated to include the first two closest to the open water mode to determine the reflection and transmission properties from open water to an ice cover (Wang and Shen, 2011). It was found that each of these two modes became dominant in an ice covers with either low or high shear modulus, respectively. For intermediate shear modulus, both modes could play a role. The results of this approximate solution were shown to be close to the exact solutions available at the thin elastic plate limit for high shear modulus cases.

In the present study, we extend the previous work to investigate wave propagation from one viscoelastic cover into another. The motivation for this study is evident: to mathematically model the wave propagation over a large expanse with varying physical conditions, a numerical method is required. All numerical methods discretize the computational domain into finite size "cells". Within each cell, average properties of variables are considered. A continuously varying ice cover is thus discretized into cells of constant thickness and material properties within each cell and abrupt changes between cells. Wave damping mechanisms contribute to the sink term within each cell. At the boundary between neighboring cells wave flux contributes to the energy transport. Both damping and flux terms are required for any numerical wave models. Transmission and reflection at cell boundaries are in fact part of the cumulative results of this process that take place at the floe scale, where all discontinuities contribute to this process. The types of floe scale discontinuities and their scattering properties are shown in Bennetts and Squire (2012). Part of the cumulative results is accounted for in the wave attenuation due to the average properties of the ice within the cell. The part due to the gradient of the ice properties within the cell is accounted for at the cell boundary. These two processes are shown schematically in Fig. 1, where the sink term has been studied as part of the dispersion relation in Wang and Shen (2010). The flux term is the focus of the present study.

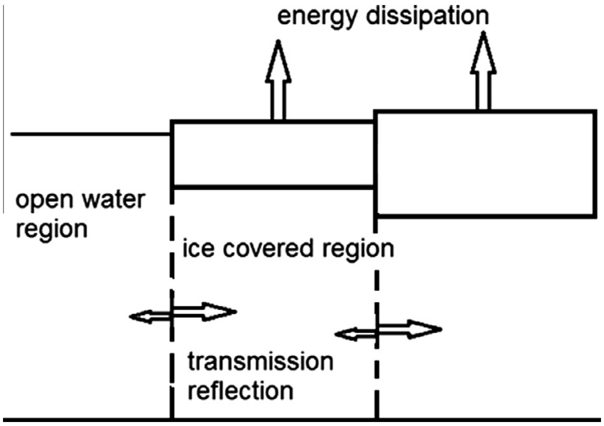

Fig. 1. Schematic of a discretized field of wave propagation into a continuous heterogeneous ice cover.

Incidentally, the same analysis provided herein may also help to expand floe scale investigations. Wave scattering theory developed by Wadhams (1973a,b) and later extensively studied by Squire and colleagues (Squire, 2007; Bennetts and Squire, 2009, 2012; Bennetts et al., 2010) considered ice floes dispersed in open water. The present work may expand these theories to situations of ice floes imbedded in a grease or brash ice field. These two different types of ice covers are shown in Fig. 2.

To determine the flux between two adjacent ice covers with different viscoelastic properties, in this study we will use the same approximate approach as given in Wang and Shen (2011). We will consider two leading modes only to determine the partition of energy of each mode for a linear monochromatic gravity wave. Our treatment of the horizontal boundary conditions will also follow the same approximation method. The organization of this paper is as follows. Section 2 briefly outlines the theoretical formulation of the viscoelastic model. In Section 3, the approximation method is presented. Section 4 gives the result of special cases to compare with previous studies for pure elastic ice covers. Section 5 discusses the characteristics of the reflection and transmission for a range of viscoelastic parameters. The summary and conclusions are given in Sections 6 and 7 respectively. A linear wave regime is assumed in this study.

Fig. 2. (Left) A photo of a broken up ice cover interspersed in open water. The narrow range of size distribution suggests a wave induced breakage. (Credit: Vernon Squire). (Right) A photo of ice floes interspersed with pancake ice. (Credit: Don Perovich).

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