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An offline implicit solver for simulating prebomb radiocarbon

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ABSTRACT

It takes several thousand years for the deep-ocean concentration of natural radiocarbon to come to equilibrium with surface fluxes, making it computationally too expensive to routinely simulate it with moderate- to high-resolution ocean models. We present an implicit solver for computing prebomb Δ^{14}C that requires the equivalent of only a few tens of model years to reach equilibrium. The solver uses a Newton–Krylov algorithm with a preconditioner based on a coarse-grained annually-averaged tracer-transport operator. Coarse-graining provides a general approach for developing preconditioners for models of increasing resolution. We implemented and tested the solver for the ocean component of the Community Earth System Model (CESM) with a nominal horizontal resolution of $1^\circ \times 1^\circ$ and with 60 vertical levels. Simulated Δ^{14}C values are in good agreement with observations at the surface and in the North Atlantic, but the deep North Pacific simulated values show a substantial bias, with prebomb radiocarbon Δ^{14}C values translating to ages that are twice the observationally based estimate. This bias is substantially larger than published simulations obtained with coarser resolution models, suggesting that increasing model resolution does not automatically improve the fidelity of the deep ocean ventilation processes. We therefore recommend that natural Δ^{14}C be used as a deep-ocean ventilation metric for critically evaluating deep ocean circulation.

1. Introduction

Assessing the fidelity of the ocean circulation in climate models is an important model development step. Models which are used to study climate trends and the evolution of the ocean's CO_2 uptake in a warming world need to be adequately tested before their results are used to influence science and national policy. Because the ocean interacts with the rest of the Earth system over a wide range of spatial and temporal scales there is no unique metric by which to judge the quality of the simulated circulation, but at long timescales the role of the deep ocean becomes increasingly important because of its capacity to store vast amounts of heat and carbon. To assess the rate at which the deep ocean communicates with the surface ocean and the atmosphere, ocean modelers have long recognized the utility of simulating natural radiocarbon (see for example the references listed in Tables 1–3). The availability of globally-gridded natural radiocarbon observations, (GLODAP, Key et al., 2004), has made radiocarbon simulations especially useful for identifying biases in the ventilation of the deep ocean.

For example, Doney et al. (2004) evaluated and contrasted radiocarbon simulations done using 13 different models as part of the Ocean Carbon Model Intercomparison Project Phase 2 (OCMIP-2) and found that errors in the simulated radiocarbon could be attributed to biases in the circulation, and that a significant part of the differences among the models could be tied to differences in sub-gridscale parameterizations. Duffy et al. (1997) and England and Rahmstorf (1999) used Δ^{14}C simulations to evaluate the effect of the Gent–McWilliams (GM) parameterization on the ventilation of the deep ocean, and found that it tended to limit the depth of convection at high latitudes. Further confirming this result, Gruber et al. (2001) found using Δ^{14}C tracer simulations, that the GM parameterization tended to give a sluggish deep circulation in their model. They showed that a 6-fold increase in the vertical diffusivity south of 50°S was needed to reduce excessive stratification and thereby improve the ventilation of the deep Pacific. Gnanadesikan et al. (2004) used Δ^{14}C simulations to evaluate the sensitivity of variations in the vertical and horizontal diffusivities in the Modular Ocean Model version 3 (MOM3). They found that both lateral and vertical mixing processes can affect the resulting Δ^{14}C distribution, but also that changes in the surface forcing can have a major impact. Butzin et al. (2005) found a strong influence of Antarctic sea ice formation on the circulation, based on Δ^{14}C simulations.

Despite its utility, radiocarbon simulations are not routinely done by climate model developers. Simulating natural radiocarbon is a considerable computational challenge because of the long timescales with which deep ocean radiocarbon equilibrates with

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Table 1
Ocean model studies before OCMIP-2 whereΔ^{14}C was used to validate or study the model. "GM?" states whether or not the Gent–McWilliams parameterization was used.

Reference	Model	Horizontal resolution lat × lon	Vertical levels	GM?	Most-depleted Pacific Δ^{14}C (‰)
Maier-Reimer and Hasselmann (1987)	Hamburg	$5^\circ \times 5^\circ$	10	N	−240
Toggweiler et al. (1989b)	GFDL Bryan–Cox	$4.5^\circ \times 3.75^\circ$	12	N	−250
Bacastow and Maier-Reimer (1990)	Hamburg HAM OCC 1	$5^\circ \times 5^\circ$	10	N	−240
Heinze and Maier-Reimer, 1991	Hamburg HAM OCC 2	$3.5^\circ \times 3.5^\circ$	11	N	−190
Maier-Reimer (1993)	Hamburg HAM OCC 3	$3.5^\circ \times 3.5^\circ$	15	N	−220
England (1995)	GFDL Bryan-Cox	$4.5^\circ \times 3.75^\circ$	12	N	−170
Duffy et al. (1997)	LLNL MOM 1.1	$3^\circ \times 3^\circ$	15	Y	−250
England and Rahmstorf (1999)	GFDL MOM	$4.5^\circ \times 3.75^\circ$	21	Y	−260

Table 2
Ocean general circulation model studies participating in OCMIP-2 whereΔ^{14}C results were submitted for the comparison. The references give descriptions of the models. If the reference does not contain the Δ^{14}C results, then the result for the most-depleted Pacific Δ^{14}C value was obtained from Orr (2002) summary of the results. If more than one value is given as the most-depleted Pacific Δ^{14}C value, the study produced the range of results given. Model with a terrain-following coordinate system is marked by †.

Reference	Model	Horizontal resolution lat × lon	Vertical levels	GM?	Most-depleted Pacific Δ^{14}C (‰)
Maier-Reimer (1993)	MPIM Hamburg	$5^\circ \times 5^\circ$	22	N	−220
Yamanaka and Tajika (1996)	IGCR	$4^\circ \times 4^\circ$	17	N	−250
Large et al. (1997)	NCAR (MOM1.1)	$1.8\text{–}0.8^\circ \times 3.6^\circ$	25	Y	−230 (Orr)
Madec et al. (1998)	IPSL OPA8† offline	$1.5^\circ \times 2^\circ$	30	Y	−190 (Orr)
Goosse and Fichefet (1999)	UL ASTR offline OCCM	$3^\circ \times 3^\circ$	20	N	−200 (Orr)
Matear and Hirst (1999)	CSIRO	$3.2^\circ \times 5.6^\circ$	21	Y	−270 (Orr)
Gordon et al. (2000)	SOC HadCM3L	$2.5^\circ \times 3.75^\circ$	20	Y	−190 (Orr)
Guilderson et al. (2000)	LLNL GFDL MOM	$2^\circ \times 4^\circ$	23	Y	−210
Follows et al. (2002)	MIT	$2.8^\circ \times 2.8^\circ$	15	Y	−200 (Orr)
Gnanadesikan et al. (2004)	PRINCE	$3.75^\circ \times 4.5^\circ$	24	Y	−220: −340

Table 3
Ocean model studies post- OCMIP-2 where Δ^{14}C was used to validate or study the model. Isopycnal models are marked with a †.

Reference	Model	Horizontal resolution lat × lon	Vertical levels	GM?	Most-depleted Pacific Δ^{14}C (‰)
Matear (2001)	MOM2	$4.25^\circ \times 3.75^\circ$	12	N	−240
Roussenov et al. (2004)	MICOM2.7†	$1.4^\circ \times 1.4^\circ$	7	N	−250
Butzin et al. (2005)	Hamburg	$3.5^\circ \times 3.5^\circ$	22	N	−220
Muller et al. (2006)	Bern3D	$10^\circ \times 10^\circ$	32	Y	−230
Galbraith et al. (2011)	GFDL MOM4p1	$0.6 - 3.4^\circ \times 3.6^\circ$	28	Y	−200
Graven et al. (2012)	POP2	$1\text{–}2^\circ \times 3.6^\circ$	25	Y	−300

the atmosphere – some models that participated in the OCMIP-2 study reported tracer equilibration times for radiocarbon of more than 4000 years (Orr, 2002). Furthermore, the computational challenge increases dramatically with increasing resolution because of Courant–Fredrichs–Lewy (CFL) stability criterion restrictions on the model time-step size. As far as we know, none of the ocean components in the current suite of IPCC-class climate models with a resolution of $1^\circ \times 1^\circ$ or higher have simulated natural radiocarbon to help calibrate the deep-ocean ventilation rate in their models.

Here we present a fast offline implicit solver for simulating natural radiocarbon in the ocean component of the Community Earth System Model (CESM). Our solver uses a Newton Krylov method and preconditioning strategy that are similar to the ones first suggested by Li and Primeau (2008) for biogeochemical tracers, but adapted for the increased memory requirements associated with the higher resolution of the CESM ocean model component, i.e. the Parallel Ocean Program version 2 (POP2), with a nominal $1^\circ \times 1^\circ$ horizontal resolution and 60 vertical levels (Smith et al., 2010). The cyclo-stationary Newton–Raphson solver and the preconditioner for the iterative Krylov-subspace linear-system solver are described in Section 6. The offline tracer-transport model and the preconditioner are based on monthly and annually averaged tracer-transport matrices, which are constructed (see Section 4), using output from a POP2 simulation, which we refer to as the parent model (Section 3). Using the new implicit solver we are able to compute the natural prebomb-Δ^{14}C equilibrium distribution by running the offline model through only 23 annual periods (Section 7.1). The formulation of the natural radiocarbon model is presented in Section 2 and the time integration scheme used by the offline model to simulate a seasonal cycle is presented in Section 5. Section 7.2 discusses the impact of neglecting the seasonal variations in ocean circulation on the simulated Δ^{14}C distribution.

We compare our Δ^{14}C simulations to the GLODAP observationally based estimates in Section 7.3, providing insight into the behavior and biases of the parent model. We also compare our results to previous radiocarbon modeling studies in Section 8 before presenting our conclusions in Section 9.

2. Governing equations for ^{14}C

Following Toggweiler et al. (1989a) we formulate our prebomb radiocarbon model in terms of the ratio $R = {}^{14}\text{C}/{}^{12}\text{C}$ expressed in arbitrary units in which the prebomb atmosphere is set to $R_{atm} = 1$,

$$\frac{\partial R}{\partial t} + \nabla \cdot (\mathbf{u}R - \mathbf{K} \cdot \nabla R) = S_{14}(R), \tag{1}$$

where $\mathbf{u}$ is the fluid velocity vector, $\mathbf{K}$ is the eddy-diffusivity tensor, and

$$S_{14}(R) = -\lambda R + \begin{cases} \mu(R_{atm} - R) & \text{if } z > -\Delta z_1, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

is the radiocarbon source-sink function including both radioactive decay with rate constant $\lambda = (8266.6\ \text{years})^{-1}$ and air–sea fluxes