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Ocean modeling on unstructured meshes

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ABSTRACT

Unstructured meshes are common in coastal modeling, but still rarely used for modeling the large-scale ocean circulation. Existing and new projects aim at changing this situation by proposing models enabling a regional focus (multiresolution) in global setups, without nesting and open boundaries. Among them, finite-volume models using the C-grid discretization on Voronoi-centroidal meshes or cell-vertex quasi-B-grid discretization on triangular meshes work well and offer the multiresolution functionality at a price of being 2 to 4 times slower per degree of freedom than structured-mesh models. This is already sufficient for many practical tasks and will be further improved as the number of vertical layers is increased. Approaches based on the finite-element method, both used or proposed, are as a rule slower at present. Most of staggered discretizations on triangular or Voronoi meshes allow spurious modes which are difficult to filter on unstructured meshes. The ongoing research seeks how to handle them and explores new approaches where such modes are absent. Issues of numerical efficiency and accurate transport schemes are still important, and the question on parameterizations for multiresolution meshes is hardly explored at all. The review summarizes recent developments the main practical result of which is the emergence of multiresolution models for simulating large-scale ocean circulation.

1. Introduction

Over the last decade the ocean circulation modeling on unstructured meshes was a subject of ongoing research, as partly highlighted in reviews by Pain et al. (2005) and Piggott et al. (2008). A number of new models has been announced, such as FVCOM (Chen et al., 2003), ICOM/Fluidity (Ford et al., 2004; Piggott et al., 2008), FESOM (Danilov et al., 2004; Wang et al., 2008), SLIM (White et al., 2008a; Blaise et al., 2010; Kärnä et al., 2013), the model by Stuhne and Peltier (2006), SUNTANS (Fringer et al., 2006), MIKE 21 & MIKE 3 Flow Model FM (http://www.mikebydhi.com), ELCIRC (Zhang et al., 2004) or SELFE (Zhang and Baptista, 2008). There are older, largely coastal or estuarine modeling efforts, such as ADCIRC (Westerink et al., 1992), QUODDY (Lynch et al., 1996), TELEMAC (Hervouet, 2000; Hervouet, 2007) or UnTRIM (Casulli and Walters, 2000). Two new projects with focus on large-scale atmosphere and ocean circulation, MPAS (http://mpas.sourceforge.net/) and ICON (www.mpimet.mpg.de/en/science/models/icon.html), also include ocean components. The numerical principles of MPAS approach are described by Thuburn et al. (2009) and Ringler et al. (2010), and the first results of MPAS-ocean simulations are very encouraging (Ringler et al., 2013). There are many more models either designed for hydrology tasks or focused solely on barotropic shallow water which are not listed here.

Unstructured meshes suggest flexibility with respect to resolving the geometry of basins. They also enable one, by locally refining computational domains, to simulate regional dynamics on a global mesh with an otherwise coarse resolution. The geometrical aspect is of utmost importance for coastal applications where computational domains involve complex-shaped coastlines and very different scales, from basin size to details of river estuaries or riverbeds. Additionally, by locally scaling the meshes as $H^{1/2}$ or $H/|\nabla H|$, where H is the water depth, one can take care of the variable surface wave speed or rapidly changing bottom topography, respectively, optimizing the mesh for simulations of tidally driven flows. The dynamical aspect is rather of interest for large-scale ocean modeling, as it offers an effective nesting approach in a global configuration free of open boundaries. The purely geometrical motivation is relevant too, but its focus shifts to places like straits, overflows or the continental break.

The research community dealing with unstructured meshes aims at providing a platform for multiresolution ocean modeling. Numerous coastal studies performed with FVCOM or ADCIRC (see their web sites for the lists of publications) vividly illustrate that the span of resolved scales can be very large (in excess of two orders of magnitude). And yet, further direct expansion from coastal toward large scales can be unpractical because the spectrum of temporal and spatial scales becomes too wide. Indeed, the mere equilibration on the global scale may take tens (if not hundreds) of years, and the fine-resolved coastal part will become an unnecessary burden. Similarly, although large-scale ocean simulations on

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global meshes with the refinement factor of about 30–50 have already been reported (see, e. g., Wang et al., 2009), it seems unlikely that this factor will be increased much further without additional measures. Given the coarse resolution of 50–100 km, such a refinement is already sufficient to reach a kilometer scale. Going beyond it may imply new physics (e.g., non-hydrostatic effects) or prohibitively large CPU cost because the time step is determined by the smallest size.

It is thus unlikely that unstructured meshes will offer a solution suited to simulate across all scales simultaneously while fully abandoning nesting. Considerations of numerical efficiency, let alone the difference in dynamics, parameterizations and mesh design, indicate that some separation between coastal and large-scale applications is likely to be preserved. This separation notwithstanding, the refinement already used in practice on unstructured meshes by far exceeds that of traditional nesting, which warrants the place for unstructured-mesh models as bridging the gap between scales and reducing the need in nesting to minimum.

Given the number of existing efforts and promises made, it seems timely to briefly summarize the achievements, questions and difficulties and draw conclusions on the further development. We do not aim at full account, leaving aside such 'high-tech' perspectives as mesh adaptivity. Instead, we try to explain what are the main difficulties as compared to structured meshes, what is already possible in practice and what should be improved, using the models known to us as an illustrating material. Our experience and hence conclusions are biased to the large-scale modeling, which is less forgiving to numerical errors than the coastal one simply because of much longer time scales. The importance of geostrophic adjustment and balance in the large-scale dynamics is the other distinguishing feature of large-scale modeling.

Speaking broadly, the main difficulty faced by models formulated on unstructured meshes lies in spurious modes maintained by discretizations. While certain spurious modes are known to occur even on regular finite-difference grids (like pressure modes on A and B grids or inertial modes on C-D grids), handling them on unstuctured meshes is more difficult. Most of staggered discretizations support branches of spurious modes which can be excited by nonlinear dynamics. Additionally, unstructured-mesh models are more expensive per degree of freedom.

Because of relatively short integration time, coastal models formulated on unsructured meshes are less vulnerable to spurious modes or to errors occurring from stabilizing them. More importantly, they offer a geometric flexibility which is difficult to achieve by other means. As a result, most of unstructured-mesh models are coastal (with ADCIRC, FVCOM, UnTRIM, SELFE and others having a long record of successful applications). The research here only seeks how to improve their already good performance or works on new functionality (like nonhydrostatic and ice components in FVCOM).

The need to handle spurious modes and the higher computational cost explain why the attempts to large-scale modeling on unstructured meshes have not always been successful or are taking too long. Unstructured-mesh large-scale ocean models now include FESOM and MPAS, with ICON working to the goal and other projects (SLIM, ICOM and FVCOM) considering it. The understanding available now is already sufficient to propose solutions that are good enough to be used in practice. However, examples showing the utility of the approach are only beginning to appear.

For convenience, Section 2 schematically explains main discretization methods used on unstructured meshes. It can safely be omitted if the reader is familiar with them. The following sections discuss the vertical coordinate, main discretization types and their properties, conservation properties, advection schemes, and reiterate on practical examples. The final sections present discussions and conclusions.

2. Main approaches

In order to facilitate further reading this section briefly sketches basic technologies of writing discretized equations on unstructured meshes — the finite element (FE) and finite volume (FV) methods. Within the FE method one distinguishes between continuous and discontinuous representations. Sometimes one uses the notion of mimetic differencing (or mimetic approach), which is related to both FE and FV methods or their combination, and places focus on mimicking the properties of continuous operators. Regular courses like Zienkiewicz and Taylor (2000), Blazek (2001) or Li (2006) contain many details.

We select an advection–diffusion equation for a tracer T to illustrate the basic approaches,

$$\partial_t T + \nabla\cdot(\mathbf{u}T - K_h\nabla T) + \partial_z(wT - K_v\partial_z T) = 0, \tag{1}$$

with $\nabla = (\partial_x, \partial_y)$ and boundary condition that tracer flux is equal to Q at the upper surface while other surfaces are 'insulated'. Here $\mathbf{u}$ and w are, respectively, the horizontal and vertical components of advecting velocity, and K_h and K_v, the diffusivity coefficients. For definiteness assume that the computational mesh is vertically extruded from a triangular surface mesh. The vertical prisms are cut into smaller prisms by a set of z-surfaces.

2.1. Continuous finite elements

According to the FE method, all fields are expanded in basis functions defined on the elements of an unstructured mesh and belonging to an appropriate functional space. We will not touch on the details of spaces here. In the simplest case the basis functions are polynomials of low order with support limited to one (usually discontinuous) or several neighboring elements (prisms). The discretized equations are obtained by projecting dynamic equations on a set of test functions. They frequently coincide with the basis functions, giving the so-called Galerkin projection. Upwind-biased test functions lead to the Petrov–Galerkin method. By its idea, the FE method resembles the spectral method.

Expand T in a set of basis functions $N_j = X_j(x,y)Z_j(z)$ defined on prismatic elements, $T = T_j(t)N_j$ (summation over repeating indices is implied if T_j is involved). Depending on the choice of functions, the index j can list mesh elements or vertices (nodes) or additional nodes in elements or on their faces. A simple example is the continuous P_1 representation (P stands for polynomial, and 1 for its degree; see section 4 for more examples). In this case X_j and Z_j equal 1 at vertex j and go linearly to zero at neighboring horizontal and vertical vertices respectively, so that $T = T_j(t)N_j$ is a bilinear interpolation which is continuous across the faces. If prisms are split into tetrahedra, the 3D linear representation becomes possible, $N_j = N_j(x,y,z)$, and the expansion T_jN_j implies a linear interpolation in three dimensions.

Next, Eq. (1) is re-written in a weak form as

$$\int (M_i\partial_t T - \mathbf{F}_h\nabla M_i - F_v\partial_z M_i)d\Omega = -\int QM_i dS, \tag{2}$$

where M_i is an appropriate test function, $\mathbf{F}_h$ and F_v are the horizontal and vertical components of fluxes and integration by parts has been performed. If $M_i = N_i$, one arrives at the Galerkin discretization

$$M_{ij}\partial_t T_j + (A_{ij} + D^h_{ij} + D^v_{ij})T_j = S_i, \tag{3}$$

where $M_{ij} = \int N_iN_j d\Omega, A_{ij} = -\int N_j(\mathbf{u}\cdot\nabla N_i + w\partial_z N_i)d\Omega, D^h_{ij} = \int K_h (\nabla N_i)(\nabla N_j)d\Omega$ and $D^v_{ij} = \int K_v\partial_z N_i\partial_z N_j)d\Omega$ are, respectively, mass, advection, horizontal and vertical diffusion matrices, and $S_i = -\int N_i Q dS$ is the source term. Note that (2) requires that N_i are at least continuous (derivatives have to be bounded). The