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Theoretical estimation of shell-side mass transfer coefficient in randomly packed hollow fiber modules with polydisperse hollow fiber outer radii

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Abstract

The expression of average shell-side mass transfer coefficient (MTC) in hollow fiber modules is theoretically deduced in this paper. This expression considers the effects of both the polydispersity of hollow fiber outer radii and the randomicity of hollow fiber distribution on the shell-side mass transfer performance for the first time. In the expression, Gaussian function is used to model the polydisperse hollow fiber outer radii, Voronoi tessellation method is used to model the random hollow fiber distribution, and the two distributions are assumed to be independent. Then, the effect of polydisperse hollow fiber outer radii on average shell-side MTC is studied in detail.

Keywords: Mass transfer coefficient; Random distribution; Polydispersity; Packing density; Hollow fiber

1. Introduction

Shell-side mass transfer of hollow fiber modules with shell and tube configuration has been studied widely in recent years. The shell-side mass transfer performance is affected by many factors [1,2]. Two of the main factors are the randomicity of hollow fiber distribution and the polydispersity of hollow fiber outer radii.

Almost all previous works about the effect of hollow fiber distribution on shell-side mass transfer have based on the assumption of identical hollow fiber outer radii. Under such assumption, commonly there are two methods to deal with the distribution of hollow fibers. One is that the hollow fiber bundle is modeled as an infinite, spatially periodic medium, i.e. the hollow fibers are assumed to be arranged in regular arrays (e.g. square or triangular) [2–6] or random in a unit cell [5,6]. The other is that Voronoi tessellation method is used to model the random distribution of hollow fibers [2,7–10].

In practice, neither inner radii nor outer radii of hollow fibers are possible to be identical because of the imperfect manufacturing process. So, the polydisperse inner and outer radii absolutely will affect lumen-side and shell-side mass transfer performance, respectively. However, in the literature, only the effect of polydisperse inner radii was investigated in detail [10–13], the effect of polydisperse outer radii seldom was focused on.

So, in this paper, the effect of polydisperse fiber radii (hollow fiber outer radii) on shell-side mass transfer is attempted to investigate theoretically in randomly packed hollow fiber modules, i.e. both the randomicity of hollow fiber distribution and the polydispersity of fiber radii are taken into account together. A theoretical average shell-side MTC expression is deduced and then the effect of polydisperse fiber radii is studied emphatically.

In the literature, a system of removing oxygen from water often was chosen to study the shell-side mass transfer performance because of its availability and simplicity [2,4,11,14,15]. Here, we make the same choice: water containing oxygen flows outside of hollow fibers, and nitrogen flows inside of hollow fibers counter-currently.

2. Theory

According to the literature [2,8–10], the following assumptions are used in this paper: (1) hollow fibers are rigid, and aligned axially but randomly arranged, i.e. the flow distribution

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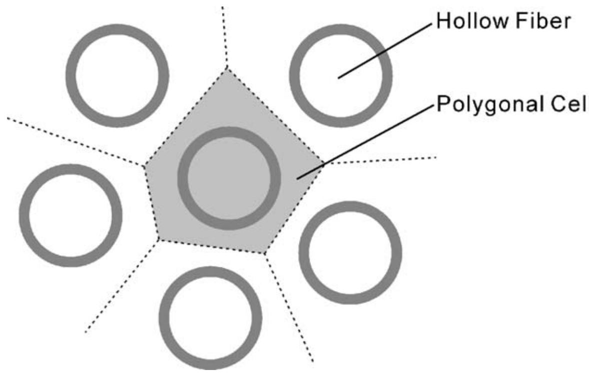

Fig. 1. Schematic representation of the module cross-section subdivision with Voronoi tessellation method.

is consistent along the module length; (2) the effect of module wall on shell-side hydraulics is ignored; (3) the fluid flow profile is fully developed and laminar; (4) the effect of shell-side inlet and outlet is ignored.

2.1. Distribution of hollow fibers

Voronoi tessellation is a mathematical method to describe the subdivision of space between randomly packed objects by drawing straight boundaries equidistant between neighboring objects, forming polygonal cells. It provides a good way to calculate the geometric characteristics of random spacing that can be used to calculate overall properties [7,16]. In this paper, Voronoi tessellation method is used to evaluate the shell-side flow distribution in a randomly packed hollow fiber module.

Considering N hollow fibers are randomly placed in the module. Using Voronoi tessellation method, the cross-section area is then subdivided into polygonal cells, and each polygonal cell is associated with one hollow fiber (Fig. 1).

The probability density distribution function of polygonal cell area can be written as below according to the literatures [2,7,11,17]:

$$f(\varphi) = \frac{s^s}{\langle\varphi\rangle^s}\frac{\varphi^{s-1}}{(s-1)!}\,\mathrm{e}^{-(s\varphi)/\langle\varphi\rangle} \tag{1}$$

$$\varphi = a - a_\mathrm{f} \tag{2}$$

where a is the polygonal cell cross-section area (including the fiber), a_f the hollow fiber cross-section area, s the number of nearest neighbor fibers, and the sign "$\langle\rangle$" denotes statistical average in mathematics. At lower packing density, $s = 4$ appears to fit the simulation better; at higher packing density, the calculated distribution agrees with $s = 6$ better, but the effect of s on results is not so evident [2,7]. So in this paper only $s = 6$ is considered for the calculation of average shell-side MTC.

Assuming that the hollow fiber distribution is independent of the hollow fiber outer radius distribution, the following equation can be obtained:

$$\langle\varphi\rangle = a_0 - \langle a_\mathrm{f}\rangle \tag{3}$$

where a_0 is the average polygonal cell cross-section area (including the fiber).

For a certain module containing N hollow fibers, the average polygonal cell cross-section area can be acquired easily:

$$a_0 = \frac{\pi R_\mathrm{M}^2}{N} \tag{4}$$

where R_M is the module inner radius. Then, the packing density can be defined as

$$\phi = \frac{\langle a_\mathrm{f}\rangle}{a_0} = \frac{N\langle r^2\rangle}{R_\mathrm{M}^2} \tag{5}$$

2.2. Polydispersity of fiber radii

According to the literature [11,12,18], the distribution of fiber radii $g(r)$ is assumed to be a Gaussian function:

$$g(r) = \frac{\mathrm{e}^{-(r-R_0)^2/(2R_0^2\varepsilon_0^2)}}{\sqrt{2\pi}R_0\varepsilon_0} \tag{6}$$

where R_0 is the average outer radius of hollow fibers, $(R_0\varepsilon_0)^2$ the variance of this distribution, and ε_0 denotes the polydispersity of fiber radii.

Obviously,

$$R_0 = \langle r\rangle = \int_0^\infty rg(r)\,\mathrm{d}r \tag{7}$$

Assuming the effective length of hollow fibers is identical and equals to L, the average hollow fiber outer surface area $\langle A\rangle$ is given by

$$\langle A\rangle = 2\pi L\int_0^\infty rg(r)\,\mathrm{d}r = 2\pi\langle r\rangle L \tag{8}$$

2.3. Flow outside of hollow fibers in polygonal cells

Using the assumptions that the entrance and exit losses are small and that flow areas are parallel and of the same length L [2,7,19,20], the shell-side pressure drop ΔP for laminar flow through each of the separate flow area is the same as the whole bundle and is calculated by

$$\Delta P = \frac{4\rho v^2 L}{d_\mathrm{h}} \tag{9}$$

where ρ is the density of water, $d_\mathrm{h} = 2\varphi/(\pi r)$ the hydraulic diameter of each polygonal cell (4 × flow area/hollow fiber circumstance), and v is the mean flow velocity in each polygonal cell. Rearrangement of Eq. (9) gives the following equation:

$$v = \sqrt{\frac{\Delta P\varphi}{2\pi r\rho L}} \tag{10}$$

Then, the average shell-side flow rate $\langle Q\rangle$ is given by

$$\langle Q\rangle = \int_0^\infty\int_0^\infty \varphi v f(\varphi)g(r)\,\mathrm{d}\varphi\,\mathrm{d}r \tag{11}$$