Food Control 29 (2013) 300–308

Contents lists available at SciVerse ScienceDirect

Food Control

journal homepage: www.elsevier.com/locate/foodcont

The impact of experiment design on the parameter estimation of cardinal parameter models in predictive microbiology

Eva Van Derlinden, Laurence Mertens, Jan F. Van Impe*

BioTeC, Chemical and Biochemical Process Technology and Control, Dept. of Chemical Engineering, KU Leuven, W. de Croylaan 46, B-3001 Leuven, Belgium

ARTICLE INFO

Article history:
Received 30 December 2011
Received in revised form
30 May 2012
Accepted 8 June 2012

Keywords:
Experimental design
Secondary cardinal parameter model
Parameter estimation
Full factorial design
Latin-square design

ABSTRACT

In predictive food microbiology, cardinal parameter models are often applied to describe the effect of temperature, pH and/or water activity on the microbial growth rate. To identify the model parameters, full factorial designs are often used, in spite of the high experimental burden and cost related to this method.

In this work, the impact of the selected experimental scheme on the estimation of the parameters of the cardinal model describing the effect of temperature, pH and/or water activity has been evaluated. In a first step, identification of a simple model describing only the effect of temperature, pH or water activity was considered. The comparison of an equidistant design and a D-optimal (based) design showed that the latter, which is based on the model's sensitivity functions, yields more realistic parameter estimates than the typical equidistant design. By selecting the experimental levels based on the sensitivity functions, a more realistic description of the behavior around optimal conditions can be obtained.

In the second step, focus was on the efficient and accurate estimation of the ten parameters of the extended cardinal model that describes the combined effect of temperature, pH and water activity on the microbial growth rate. Again, equidistant level selection is compared to a D-optimal (based) experimental design. In addition, a full factorial and a Latin-square approach are evaluated. From the simulation case studies presented, it can be stated that all parameters can be equally well defined from an equidistant design as from a D-optimal-based design. In addition, reducing the experimental load by constructing a Latin-square design does not hamper the parameter estimation procedure. This work confirms the observation of a previous study, i.e., for complex cases a Latin-square design is an attractive alternative for a full factorial design as it yields equally accurate and reliable parameter estimates while reducing the experimental workload.

1. Introduction

In predictive microbiology, mathematical models are developed that can quantify the microbial evolution in food products or food processing environments. The development of a mathematical model follows the general model building procedure, which encloses three basic steps (Asprey & Macchietto, 2000; Tarantola, 2005). Initially, relevant a priori knowledge is collected and a set of independent variables (e.g., physical quantities) is determined which defines the investigated system (parameterization of the system). Next, one or more mathematical model structures are proposed that are able to predict the system's behavior (forward modeling). A precondition is that the models are structurally identifiable, i.e., all model parameters can be identified uniquely. In the third step, the predictive quality of the models is improved by adjusting the intrinsic model parameters to the actual measurements (parameter estimation). Subsequently, the validity of its structure and parameter estimates is tested against new experimental data. When the descriptive quality of the model is insufficient, an adaptation of the model structure and/or parameter estimates is needed. As such, model structure selection and parameter estimation are combined in an iterative model building

---

Abbreviations: DOE, design of experiment; OED/PE, optimal experiment design for parameter estimation; FF, full factorial design; FF-EQ, full factorial design with equidistant distribution of the experimental levels; FF-OED, full factorial design with a D-optimal-based experimental level distribution; LS, Latin-square design; LS-EQ, Latin-square design with equidistant distribution of the experimental levels; LS-OED, Latin-square design with a D-optimal-based experimental level distribution; SE, standard error of secondary model parameter estimates; StDev, standard deviation of plate count measurements.

* Corresponding author. Chemical and Biochemical Process Technology and Control (BioTeC), Dept. of Chemical Engineering, KU Leuven, W. de Croylaan 46, B-3001 Leuven, Belgium. Tel.: +32 16 32 14 66; fax: +32 16 32 29 91.

E-mail address: jan.vanimpe@cit.kuleuven.be (J.F. Van Impe).

0956-7135/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.foodcont.2012.06.018

cycle. Finally, a model is obtained that agrees with reality as closely as possible.

Both the selection of an appropriate model structure and the identification of accurate model parameters are data-driven processes, i.e., the efficiency and accuracy of these procedures are determined by the quality of the experimental data. Experimental data should enclose sufficient information with respect to the studied system. As a result, a well-thought experimental scheme can significantly improve model building and parameter estimation.

In the last decennia, a myriad of predictive models has been developed that can be applied to describe the effect of environmental conditions (e.g., temperature and pH) on the microbial growth rate. For some models, it is assumed that the model structure is valid for a wide series of food pathogens and food spoilage microorganisms. Models fitting this description are, for instance, square-root-type models and cardinal parameter models. For both model types, a first model was constructed to describe the effect of temperature on the microbial growth rate (see Ratkowsky, Olley, McMeekin, & Ball, 1982; Rosso, Lobry, & Flandrois, 1993). Afterward, both square root and cardinal parameter models were extended to consider multiple environmental factors, based on the gamma concept. In this concept, it is assumed that different influencing factors act independently such that they can be used in a multiplicative way. When applying these model types, model structures are taken as correct and the focus is, therefore, on the determination of accurate model parameters.

In predictive microbiology, experimental schemes designed for model parameter estimation are usually chosen arbitrarily. When focusing on a single environmental factor, most often, a set of equidistant constant levels is selected. When considering multiple environmental factors, often full factorial designs are used in spite of the high experimental workload. These options are still more rule than exception despite the fact that it is proven that a specific design can highly improve the parameter estimation efficiency and accuracy (see, e.g., Balsa-Canto, Alonso, & Banga, 2008; Bernaerts, Servaes, Kooyman, Versyck, & Van Impe, 2002; Mertens, Van Derlinden, & Van Impe, 2012; Van Derlinden, Bernaerts, & Van Impe, 2008, 2010). In all of these cases, two different mathematical techniques/approaches are applied to increase and/or optimize the information contained in a limited series of experiments.

(1) Presuming model validity, the mathematical technique of **optimal experiment design for parameter estimation** (OED/PE) forms an excellent starting point for the selection of a small set of highly informative, static and/or dynamic experiments, resulting in unique and accurate parameter estimates. When applying dynamic experiments, this approach also guarantees parameter estimates which are valid under varying, more realistic conditions.
(2) **Design of experiments** (DOE) is an approach based on statistical aspects that enables the determination of the relation between (environmental) factors, their interactions and statistical properties. Instead of studying the system at single levels for one environmental factor at a time, more information can be collected in a full factorial experiment design in which all combinations of the selected discrete levels of the different explanatory variables are considered. The number of static experiments can be further reduced by a well-thought selection of these levels that unravel the most information about the studied system.

For low complexity cases, full factorial designs (FF) are generally considered as the most appropriate design. However, the FF approach is rather conservative, and very labor-intensive and costly, certainly when a high number of variables and/or an extended range of levels is considered. Recently, Mertens et al. (2012) showed that a Latin-square design (LS) is an advisable alternative to full factorial design for complex cases, i.e., when a high number of environmental factors and/or a high number of levels is considered. Second order experimental designs like Box–Behnken and Central Composite Design are specifically useful to identify curvatures and are, therefore, best applied for response surface modeling and better not used for square-root-type models (Mertens et al., 2012).

In this work, the focus is on the accurate estimation of the parameters of a widely used extended cardinal parameter model that describes the effect of temperature, pH and water activity on the microbial growth rate. This model includes ten parameters: $T_{\min}$, T_{opt}, $T_{\max}$, $\mathrm{pH}_{\min}$, $\mathrm{pH}_{\mathrm{opt}}$, $\mathrm{pH}_{\max}$, a_{wmin}, a_{wopt}, a_{wmax} and μ_{opt}. In this manuscript, the impact of the selected experimental design on the estimation of these parameters is evaluated. In a first step, the three factors (temperature, pH and water activity) are considered separately, i.e., it is evaluated how the model describing the effect of only one factor can be identified the most efficiently. Hereto, an equidistant design is compared to a D-optimal (based) design. Next, this knowledge is used to build a well-founded (D-optimal-inspired) design to identify all ten parameters of the extended cardinal model simultaneously. This approach is compared to a typically equidistant design. For both approaches, a full factorial design and a Latin-square design are applied.

The results presented in this manuscript are an extension on the work of Mertens, Van Derlinden, and Van Impe (2011) on the effect of the experimental design on the estimation of square-root-type model parameters, as presented at the 7th International Conference on Predictive Modeling of Food Quality and Safety.

2. Materials and methods

2.1. The secondary model under study

The secondary model under study is the cardinal parameter model that describes the effect of temperature, pH and water activity on the microbial growth rate:

$$\mu_{\max} = \mu_{\mathrm{opt}} \cdot \gamma(T) \cdot \tau(\mathrm{pH}) \cdot \rho(a_{\mathrm{w}}) \tag{1}$$

with

$$\gamma(T) = \frac{(T - T_{\max})(T - T_{\min})^2}{(T_{\mathrm{opt}} - T_{\min})\left[(T_{\mathrm{opt}} - T_{\min})(T - T_{\mathrm{opt}}) - (T_{\mathrm{opt}} - T_{\max})(T_{\mathrm{opt}} + T_{\min} - 2T)\right]} \tag{2}$$