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Theoretical analysis of particle trajectories and sieving in a two-dimensional cross-flow filtration system

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Abstract

Particle deposition and fouling are critical factors governing the performance of microfiltration systems. Particle trajectories in cross-flow filtration were evaluated by numerical integration of the Langevin equation accounting for the combined effects of electrostatic repulsion, enhanced hydrodynamic drag, Brownian diffusion, inertial lift and van der Waals attraction. The membrane remains completely free of particles below a critical filtration velocity due to the electrostatic repulsion between the charged particles and the charged membrane. This critical flux increases with increasing surface potential and decreasing ionic strength due to the increase in electrostatic repulsion. The critical flux also increases with increasing wall shear rate due to the reduction in residence time over the pore. Brownian motion provides a random character to the particle trajectories, allowing particles to enter the pores even at operation below the critical flux. Particle transmission increases with increasing filtrate flux and ionic strength, and decreases with increasing particle size, wall shear rate and electrostatic potential.

Keywords: Particle trajectory; Critical flux; Sieving; Cross-flow filtration; Microfiltration

1. Introduction

Pressure-driven membrane filtration is currently used for the separation of particles and cells from liquid suspensions in many areas, such as waste treatment [1], food processing [2], biotechnology [3] and water purification [4]. Although normal flow filtration is often used for very dilute suspensions, the treatment of more concentrated suspensions is conducted using cross-flow filtration, which is also known as tangential flow filtration. The cross-flow configuration, in which bulk flow is parallel to the filtering membrane and perpendicular to the filtrate flow, minimizes accumulation of retained species at the membrane surface. This provides much larger filtration rates than can be attained in normal flow (dead-end) filtration. During cross-flow filtration, suspended particles are transported to the membrane surface by the filtrate flux, with the particle behavior in the region adjacent to the membrane surface playing a critical role in understanding membrane fouling, particle deposition and particle retention.

The fluid velocity profiles in cross-flow filtration systems have been studied by a number of investigators, and an extensive review of the literature on flow in porous ducts is provided by Belfort [5]. The majority of these studies have been limited to low Reynolds number flows, with the velocity evaluated by solution of the steady-state Navier–Stokes equations typically using a perturbation method assuming that the wall is uniformly permeable (i.e., neglecting the presence of discrete pores). Berman [6] developed the first quantitative solution for laminar flow in a flat channel with two porous walls. This approach was extended to higher Reynolds numbers by Gupta and Levy [7] using a power series expansion for the velocity in terms of Re_{w}. Singh and Laurence [8] examined the effect of a slip velocity at the porous wall, and Kleinstreuer and Paller [9] investigated the behavior when the permeation rate varies with axial position.

Particle trajectories in cross-flow filtration are considerably more complex due to the additional hydrodynamic and external forces exerted on the particle. Most studies of membrane systems have focused on the effects of inertial lift forces arising from hydrodynamic interactions on a particle moving near a solid boundary. These forces give rise to the well-known tubular-pinch effect, which has been extensively studied both experimentally and theoretically [10–14].

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Belfort and Nagata [15–17] evaluated the trajectory of individual particles during cross-flow microfiltration by solving the Navier–Stokes equations including the inertial terms. The membrane was treated as a uniformly porous boundary, without distinct pores. Particular emphasis was placed on calculating the effects of inertial lift and transmembrane flow (suction) on particle motion at realistic particle Reynolds numbers. Results demonstrated that large particles are transported away from the membrane surface due to lift forces, allowing one to operate the membrane device at high filtration velocities without significant fouling or polarization. Small particles tend to deposit on the membrane surface since the drag force due to filtration is greater than the inertial lift.

Schmitz et al. [18] evaluated the importance of vortex formation within an individual pore at high filtration velocities on the rate of particle deposition. The results were used to evaluate the thickness of the fluid region in which all particles were captured on the membrane surface or enter the pore. Calculations were also provided for the impact location and capture efficiency for both two- [19] and three-dimensional [20] systems. Kleinstreuer and Chin [21] studied particle deposition and fouling layer growth in cross-flow filtration using particle trajectory analysis. Calculations were performed accounting for the interaction between particles, the fluid flow rate, cake formation and flux decline. Sethi and Wiesner [22] proposed a unified transient model for cross-flow filtration based on the formulation originally developed by Romero and Davis [23] including the effects of Brownian diffusion, inertial lift and shear-induced diffusion.

Although these studies have provided important insights into particle motion in cross-flow membrane filtration, none of these investigations have considered the combined effects of electrostatic interactions, enhanced hydrodynamic drag, van der Waals forces, Brownian diffusion and inertial lift on particle motion in cross-flow filtration. In order to obtain a more fundamental understanding of particle behavior in cross-flow filtration, the theoretical framework developed by Kim and Zydney [24] was extended to consider the effects of cross-flow filtration. FLUENT simulations were performed to evaluate the importance of the different forces on particle transport, including the effects of solution properties and particle size on the particle trajectories and retention characteristics during cross-flow filtration.

2. Theoretical development

2.1. Fluid flow

Model simulations were performed for a two-dimensional channel formed between two porous membranes as shown schematically in Fig. 1. The distance between the membranes is H and the channel length is L_f. The membrane itself is composed of a periodic array of slit-shaped pores with the entry to each pore defined by a cylinder of radius b. Each pore has a half width of r_0 and the spacing between the pores is L_p. The Reynolds number in the channel is assumed to be less than one corresponding to creeping flow.

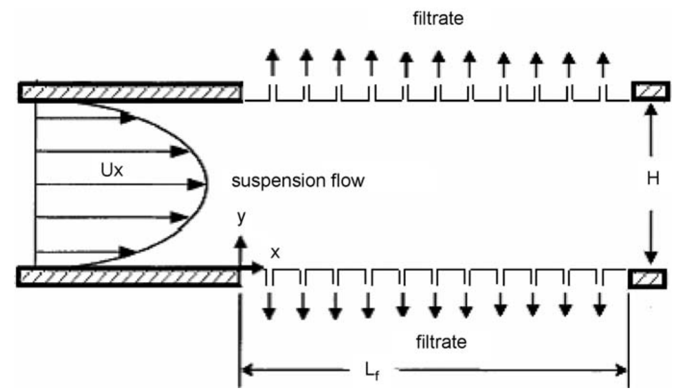

Fig. 1. A schematic diagram of a two-parallel-plate filtration system.

The velocity profiles in the particle-free system are governed by the steady-state Navier–Stokes equations for incompressible laminar creeping flow:

$$-\nabla p + \mu \nabla^2 \mathbf{u} = \mathbf{0} \tag{1}$$

$$\nabla \cdot \mathbf{u} = \mathbf{0} \tag{2}$$

where $\mathbf{u}$ is the velocity vector and p is the local hydrostatic pressure. The boundary condition at the entrance to the porous region of the channel was taken as the fully developed velocity profile,

$$u_x = 6u_s \left[\left(\frac{y}{H} \right) - \left(\frac{y}{H} \right)^2 \right] \tag{3}$$

where u_s is the average cross-flow velocity at the channel inlet. The inlet pressure is also assumed to be uniform at a value P_0.

Symmetry conditions are imposed at the channel centerline, $y = H/2$. No-slip and no normal flow conditions are imposed along the entire membrane surface, including the walls of the pore. The pressure at the exit to each pore is assumed to be uniform at a value of P_f. Note that P_f corresponds to the filtrate pressure only if the simulations are performed over the entire pore length (typically 1 μm for the skin layer of an asymmetric membrane and around 100–250 μm for a homogeneous membrane). The feed-side pressure varies by about 1% (or less) over the length of the 11-pore system at the flow rates examined in this study.

2.2. Particle trajectories

Particle trajectories were calculated for spherical particles located in the region above the membrane by numerical integration of the Langevin equation:

$$m_p \frac{d\mathbf{u}_p}{dt} = 6\pi\mu a[K_p \mathbf{u}_p - K_f \mathbf{u}_f] + F_E + F_B + F_L + F_{VDW} \tag{4}$$

where $\mathbf{u}_p$ is the velocity vector for the particle, $\mathbf{u}_f$ the fluid velocity at the location of the particle center, which is evaluated from the solution of the Navier–Stokes equations in the absence of any particles, a is the particle radius, and m_p is the particle