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An analytical approach to the gas pressure drop in hollow fiber membranes

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Abstract

The gas pressure drop in the bore of hollow fiber membranes has attracted much attention. Numerical methods are generally employed to deal with such complex problems. However, numerical methods cannot provide a clear picture on how the pressure drop is affected by the physical properties of fibers (e.g., fiber inner diameter, fiber length, and intrinsic permeance) and the operating conditions (e.g., downstream pressure). This article attempts to elucidate the interaction mechanism based on an analytical approach. The pressure drop in hollow fibers is formulated by an elliptic integral function, and the effectiveness factor of the transmembrane pressure is proposed to characterize the effect. A dimensionless parameter is obtained, which contains all the dominant factors responsible for generating the pressure drop in hollow fibers. It is shown that the dimensionless parameter serves as the indicator of the significance of the effect of the pressure buildup in hollow fibers. To some degree, this parameter is also a convenient tool for rationalizing hollow fiber modules design.

Keywords: Hollow fiber membrane; Pressure drop; Analytical approach; Effectiveness factor; Gas permeation

1. Introduction

Membranes have played an important role in gas separations [1–3], where most of the membranes are in the hollow fiber configuration. The commercial success based on hollow fiber permeators was first achieved by Dupont [4], which was followed by Monsanto [5–7], Dow [8–10], Union Carbide [10], and Ube Industries [10–12], etc. It is generally admitted that these commercial successes resulted from the distinct features of hollow fibers, which are self-support, high-pressure resistance and high compactness.

It is well known that the compactness of hollow fiber separators depends largely on the size of individual hollow fibers. Table 1 presents the estimated compactness data of various modules of hollow fibers with different outer diameters, and with different packing densities. The advantage of using small fibers is justified by the corresponding increased compactness. However, according to the Hagen–Poiseuille equation, hollow fibers with small inner diameters can result in big pressure drops in the bore of the fibers. As a result, the driving force for gas permeation through the fibers can be significantly reduced, depending on how small the inner diameter of the fibers is. Obviously, the tradeoff between the compactness and the unfavorable pressure drop should be reasonably compromised so as to maximize the advantages of hollow fiber membranes in applications. As a first step toward this objective, the pressure drop in hollow fiber membranes should first be investigated.

Studies involving the gas pressure drop in hollow fibers have been well documented [13–20]. The central effort of these studies was devoted to formulating numerical solutions to a set of equations (i.e., the permeation equations, the mass balance equations and the Hagen–Poiseuille equation), and to elucidating the interdependence of some important design parameters (e.g., the recovery of product gas, the purity of product gas). The effect of the pressure buildup on gas permeation through the membranes, though considered in computations, was usually ignored in discussion. Most importantly, due to the “implicit” nature of the numerical methods, the

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Table 1
The relationship between the compactness and the sizes of hollow fibers

Fiber diameter (μm)	Packing density[a] (%)	Compactness[b] (10^3 m^2/m^3)
150	50	6.65
	40	5.35
120	40	8.35
80	40	12.50
40	35	21.30
Hollow fiber	–	6.67–13.33[c]
Spiral wound	–	0.67–1.17[c]
Plate-and-frame	–	0.33–0.50[c]

[a] Packing density refers to the relative volume occupied by fiber bundle in a separator.
[b] The volume of the shell wall is assumed to be equal to the inner housing space in estimating the compactness data.
[c] The data converted from Reference [10].

mechanism of how the geometric, and permeating properties of hollow fibers, as well as the operating conditions exercise their respective effect on the pressure drop in hollow fibers was not yet well elucidated. An analytical treatment of this issue, intended for explicitly disclosing the interaction mechanism is thus necessary.

Some analytical effort was paid to modeling liquid permeation through permeable hollow fibers. Middleman [21] studied the liquid flow in hollow fiber filtration membranes using the perturbation approach. The equation governing the pressure drop in porous hollow fibers was obtained in the following form:

$$p_g(z) = C_1\exp(\alpha z) + C_2\exp(-\alpha z), \quad \alpha = \frac{4}{R}\sqrt{\frac{k\mu}{R}}$$

where $p_g(z)$ stands for the gauge pressure of the fluid in hollow fibers, z the location variable, k the permeability of the porous membrane, μ the viscosity of the fluid, R the inner radius of the fibers, and C_1 and C_2 are two constants, which can be determined by two boundary conditions, i.e., the pressure, and the flow rate at the fiber inlet. Munson-McGee [22] approximated Middleman's solution using a parabolic fitting: $p_g(z) = a + bz + cz^2$, since he found that the curve of $p_g(z)$ versus z could be well represented by a parabolic function. The three constants a, b, and c in the equation were determined by the two boundary conditions at the fiber inlet, and the mass conservation equation over the length of the fibers. The fault of this treatment is that it contains three constants so that the two boundary conditions at the inlet are not enough to determine the function. It is thus impossible to predict the filtration process based on the inlet conditions and the permeation properties of the hollow fibers. Due to the compressibility of gases, the equation governing the pressure drops of liquids in permeable fibers must not apply for gases.

In this study, the gas pressure drop in hollow fibers is formulated by an elliptic integral function, and the feature of the gas pressure drop in hollow fibers was accordingly investigated. Fundamentally speaking, the presence of the pressure drop in hollow fibers deceases the driving force for gas permeation through the hollow fibers. The effectiveness factor of the transmembrane pressure/the driving force is thus proposed to characterize the effect. A dimensionless parameter was obtained through mathematical treatment. It contains all the dominant factors leading to the pressure drop in hollow fibers, and analysis shows that the dimensionless parameter is an indicator of the significance of the effect of the pressure drop, and based on the parameter, the reasonableness of hollow fiber module design can be easily tested. The significance of the rational design of hollow fiber modules was also briefly elucidated with the effect of the pressure drop on the ideal separation factor of hollow fiber modules using some industrially important gas pairs.

2. Theoretical

2.1. Gas pressure drop in hollow fibers

Fig. 1 is a schematic diagram of a hollow fiber. Both of the fiber ends are potted in tube sheets, with one end open to provide outlet for the permeate, which flows in the bore of the fiber. Due to the small inner diameter of the fiber, a pressure drop of the permeate can be rendered. According to the literature [15–19,23], the Hagen–Poiseuille equation as shown in Eq. (1) is eligible for governing gas flow in the bores of cylindrical hollow fibers with some reasonable assumptions (e.g., idea gas, laminar flow, low radial flow relative to the axial flow):

$$\frac{dp^2}{dl} = -\frac{256RT\mu}{\pi D_i^4}N \quad (1)$$

where p refers to the local pressure of the permeate, l the distance of a random position to the sealed end of the fiber, D_i the inner diameter of the fiber, N the molar flow rate of the permeate, μ the viscosity of the permeate, R the universal gas constant, and T is the absolute temperature of the permeate.

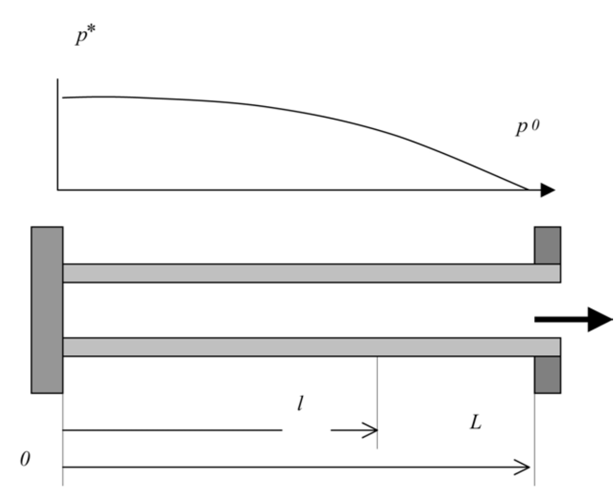

Fig. 1. The schematic representation of a potted hollow fiber and the profile of pressure drop across the fiber length.